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USE OF THE CLASS OF EQUIRADIAL EXPERIMENTAL DESIGNS
IN RESPONSE SURFACE METHODOLOGY

A THESIS

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The Faculty of the Division of Graduate
Studies and Research

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Claude K. Hudson

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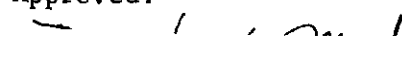
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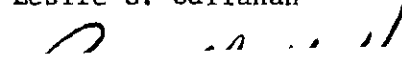
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SUMMARY

This thesis presents a study of the class of equiradial experimental designs for use in response surface methodology. The research surveys the field of response surface methodology and the use of second order experimental designs in fitting and optimizing response surfaces.

The construction of second order experimental designs is studied, with particular emphasis on the class of equiradial designs. The concept of rotatability and other design criteria are developed and illustrated in the construction of equiradial designs.

A representative selection of two factor equiradial designs is constructed and applied to a variety of known response surfaces under various controlled conditions. A similar application is made of several standard central composite designs. The results for all of these designs are collected into several logical groups for analysis. Conclusions drawn from this analysis are that the best performance is provided by those equiradial designs derived from equiradial pentagons and hexagons, in particular, by uniform precision designs so derived. Pentagon and hexagon derived designs are also seen to generally outperform central composite designs.

CHAPTER I

INTRODUCTION

1.1 Response Surface Methodology

Response surface methodology is a collection of mathematical and statistical techniques first produced by Box and Wilson (4). The goal of response surface methodology is to find the optimum of a response y , where

$$y = \phi(x_1, x_2, \dots, x_k) \quad (1.1)$$

is a function of several variables x_1, x_2, \dots, x_k . The true functional relationship ϕ is unknown and successive observations on y are obscured by random error.

In general, response surface methodology approximates the relationship (1.1) by a regression model of the form

$$y = \underline{x}'\underline{\beta} + \epsilon, \quad (1.2)$$

where \underline{x} is a vector of the independent variables, $\underline{\beta}$ is a vector of regression coefficients, and ϵ is the random error associated with the observation on y . The procedure is sequential in nature, usually first applying a linear regression model to fit a first order response equation (surface) until a lack of fit is detected and then fitting a higher order response surface, usually of second order. The form of the approximating function is usually a polynomial such as

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \dots + \beta_k x_k + \epsilon \quad (1.3)$$

for the first order approximation, and

$$y = \beta_0 + \sum_i \beta_i x_i + \sum_i \beta_{ii} x_i^2 + \sum_{i < j} \sum_j \beta_{ij} x_i x_j + \epsilon \quad (1.4)$$

for the second order approximation.

Fitting a second order regression model requires the use of a second order experimental design. The most widely known and used classes of second order experimental designs are factorial and central composite designs. Another, less widely used, class of second order experimental designs is the equiradial class. By far, the majority of studies on construction and performance of second order experimental designs have dealt primarily with central composite designs. The purpose of this study is to investigate the class of second order equiradial designs.

1.2 Objectives

The objectives of this thesis are:

1. To investigate the properties, construction, and use of the class of equiradial designs.
2. To analyze the effectiveness of designs from this class in comparison to central composite designs.
3. To draw conclusions regarding this class of designs.

These objectives will be accomplished by surveying response surface methodology and second order experimental designs, with particular attention to the class of equiradial designs. A representative group of equi-

radial designs will then be constructed and applied to several known response surfaces under various controlled conditions. Central composite designs will be applied in the same manner. The results will then be compared and evaluated, and conclusions will be drawn.

1.3 Assumptions

The following assumptions are made:

1. In the testing of various designs on the surfaces to be discussed later, it is assumed that a region has already been encountered where there is a "lack of fit" for the first order model. At this point, fitting of a second order model is deemed appropriate.

2. The equiradial designs from pentagon (5 vertices) to decagon (10 vertices) inclusive are selected as representative of the class of equiradial designs.

CHAPTER II

RESPONSE SURFACE METHODOLOGY

2.1 General

Response surface methodology seeks to optimize a response by approximating the unknown functional relationship between the response and its independent variables and then optimizing the approximating function. This approximation makes use of the general linear regression model

$$\underline{y} = X \underline{\beta} + \underline{\epsilon} , \quad (2.1)$$

where \underline{y} is a vector of response observations, X is a matrix in which the i^{th} row represents the levels of the independent variables associated with the i^{th} response observation, $\underline{\beta}$ is the vector of regression coefficients, and $\underline{\epsilon}$ is a vector in which the i^{th} element represents the random error associated with the i^{th} observed response. In general, \underline{y} is $(N \times 1)$, X is $(N \times p)$, $\underline{\beta}$ is $(p \times 1)$, and $\underline{\epsilon}$ is $(N \times 1)$.

It should be noted that this model is linear only in the β 's and is therefore applicable to any order response surface. Consider the first order response model

$$y = \beta_0 + \sum_{i=1}^k \beta_i X_i + \epsilon . \quad (2.2)$$

Here the $\underline{\beta}$ vector is

$$\underline{\beta}' = [\beta_0, \beta_1, \dots, \beta_k] ,$$

and a typical row of the X matrix is

$$\underline{x}_i = [1, x_{i1}, \dots, x_{ik}] .$$

The second order response model is

$$y = \beta_0 + \sum_{i=1}^k \beta_i x_i + \sum_{i=1}^k \beta_{ii} x_i^2 + \sum_{i < j} \beta_{ij} x_i x_j + \epsilon . \quad (2.3)$$

In this case the $\underline{\beta}$ vector is

$$\underline{\beta}' = [\beta_0, \beta_1, \dots, \beta_k, \beta_{11}, \dots, \beta_{kk}, \beta_{12}, \dots, \beta_{(k-1)k}] ,$$

and a typical row of the X matrix is

$$\underline{x}_i = [1, x_{i1}, \dots, x_{ik}, x_{i1}^2, \dots, x_{ik}^2, x_{i1}x_{i2}, \dots, x_{i(k-1)}x_{ik}] .$$

In fitting the model we wish to estimate the $\underline{\beta}$ vector in such a manner as to minimize the sum of squares of the error. Our problem then is

$$\min L = \min \sum_{i=1}^k \epsilon_i^2 = \min \underline{\epsilon}' \underline{\epsilon} \quad (2.4)$$

Solving equation (2.1) for $\underline{\epsilon}$ gives

$$\underline{\epsilon} = \underline{y} - X \underline{\beta} . \quad (2.5)$$

Substituting equation (2.5) into equation (2.4) and simplifying yields

$$\min L = \min [\underline{y}'\underline{y} - 2\underline{\beta}' \underline{X}' \underline{y} + \underline{\beta}' \underline{X}'\underline{X} \underline{\beta}] .$$

Taking the partial derivative of L with respect to $\underline{\beta}$ and setting it equal to zero yields

$$\frac{\partial L}{\partial \underline{\beta}} = - 2\underline{X}'\underline{y} + 2(\underline{X}'\underline{X})\hat{\underline{\beta}} = 0 .$$

Solving this for $\hat{\underline{\beta}}$ provides the least squares estimator of $\underline{\beta}$,

$$\hat{\underline{\beta}} = (\underline{X}'\underline{X})^{-1} \underline{X}'\underline{y} . \quad (2.6)$$

A response surface analysis usually begins with a first order investigation and proceeds in this manner until a lack of fit is encountered and a near optimum condition is suspected.

Numerous first order optimization methods are available. Among the most commonly used methods are steepest ascent, univariate, factorial, and random search. The steepest ascent and univariate are sequential search methods. As its name implies, the steepest ascent method determines the steepest gradient in a small region and ascends in that direction until another gradient determination is necessary. In the univariate method, one factor at a time is varied over the region of interest while all others are held constant. This is repeated until no further improvement in response is achieved. The factorial method uses a single factorial experiment for the entire region of interest. In the random method,

trials are made at random throughout the region of interest.

Brooks (5) made a comparative study of these four first order methods. He tested each method on the four surfaces shown in Figures 1(a) through 1(d). The results of his study indicated that the sequential methods performed best and that the steepest ascent method performed better than the univariate method, on the average.

2.2 Second Order Investigation

Consider the second order fitted response model

$$\hat{y} = \hat{\beta}_0 + \sum_{i=1}^k \hat{\beta}_i x_i + \sum_{i=1}^k \hat{\beta}_{ii} x_i^2 + \sum_{i < j} \hat{\beta}_{ij} x_i x_j .$$

This may be rewritten in matrix form as

$$\hat{y} = b_0 + \underline{x}' \underline{b} + \underline{x}' B \underline{x} , \quad (2.7)$$

where

$$\begin{aligned} \underline{x}' &= [x_1, x_2, \dots, x_k] , \\ b_0 &= \hat{\beta}_0 , \\ \underline{b}' &= [\hat{\beta}_1, \hat{\beta}_2, \dots, \hat{\beta}_k] , \end{aligned}$$

and

$$B = \begin{bmatrix} \hat{\beta}_{11} & \frac{1}{2}\hat{\beta}_{12} & \dots & \frac{1}{2}\hat{\beta}_{1k} \\ (s_{y_{mm}} & \hat{\beta}_{22} & \dots & \frac{1}{2}\hat{\beta}_{2k} \\ \text{symmetric) } & \dots & \dots & \dots \\ & & \hat{\beta}_{kk} \end{bmatrix} .$$

To find the factor combination which optimizes y we differentiate

equation (2.7) with respect to \underline{x} and set the results equal to zero. This yields

$$\frac{\partial y}{\partial \underline{x}} = \underline{b} + 2B\underline{x} = 0 ,$$

or

$$\underline{x}_0 = -\frac{1}{2}B^{-1}\underline{b} , \quad (2.8)$$

the estimated point of optimum response. Substituting equation (2.8) into equation (2.7) and simplifying gives the predicted optimum response:

$$y_0 = b_0 + \frac{1}{2}\underline{x}_0'\underline{b} \quad (2.9)$$

The stationary point \underline{x}_0 is, of course, not necessarily a maximum. In fact it may be either a maximum, a minimum, or a saddle point. To determine the nature of the surface at the stationary point one must conduct a canonical analysis. The canonical analysis translates the variable axes to the stationary point and then rotates the axes to correspond to the principal axes of the contour system. Substituting the transformation $\underline{z} = \underline{x} - \underline{x}_0$ into equation (2.7) and simplifying yields

$$\hat{y} = \hat{y}_0 + \underline{z}'B\underline{z} , \quad (2.10)$$

the response in terms of the \underline{z} variables. This accomplishes the translation of the axes to the stationary point. Setting $\underline{z} = M\underline{w}$, equation (2.10) may be expressed as

$$y = y_0 + \underline{w}'M'B M\underline{w} = y_0 + \sum_{i=1}^k \lambda_i w_i^2 , \quad (2.11)$$

where \underline{w} represents the principal axes of the contour system, M is the orthogonal transform of \underline{w} onto \underline{z} , and

$$M'BM = \begin{bmatrix} \lambda_1 & & & \\ & \lambda_2 & & 0 \\ & & \ddots & \\ 0 & & & \lambda_k \end{bmatrix}$$

where the λ_i are the eigenvalues of the matrix B .

Inspection of the λ_i 's will reveal the nature of the surface at the stationary point. From equation (2.11) it is clear that, if all λ_i are negative, then a move along any w axis away from the stationary point (origin of the w axes) will reduce the response, and hence we have a maximum. Similarly, if all λ_i are positive we have a minimum, and if the λ_i have mixed signs we have a saddle point.

2.3 Selection Criteria for Second Order Designs

In order to conduct a second order analysis, a second order experimental design must be used to estimate the model parameters. Several performance criteria have been developed for the selection of response surface designs. These will be discussed in this section, but first it is useful to introduce the concepts of design moments and precision.

2.3.1 Design Moments and Precision

The design moments are expressed in terms of the levels of the design variables. Consider the design matrix

$$D = \begin{bmatrix} x_{11} & \dots & x_{k1} \\ \dots & \dots & \dots \\ x_{1N} & \dots & x_{kN} \end{bmatrix}$$

where the i^{th} row represents the variable levels to be set for the i^{th} observation. In general the design moments are

$$[1^{\alpha_1} 2^{\alpha_2} \dots k^{\alpha_k}] = \frac{1}{N} \sum_{u=1}^N x_{1u}^{\alpha_1} x_{2u}^{\alpha_2} \dots x_{ku}^{\alpha_k} . \quad (2.12)$$

The order of the moment is equal to the sum of the α_i 's. The moment is even if all α_i 's are even and odd if any α_i is odd. If only one α_i is nonzero, then the moment is said to be a pure moment, otherwise it is mixed.

The moments of a d^{th} order experimental design are of order through $2d$. Hence a second order design will have design moments of order through four. These include the second pure moment

$$[ii] = \frac{1}{N} \sum_{u=1}^N x_{iu}^2 , \quad (2.13)$$

the fourth pure moment

$$[iiii] = \frac{1}{N} \sum_{u=1}^N x_{iu}^4 , \quad (2.14)$$

and the fourth mixed even moment

$$[iijj] = \frac{1}{N} \sum_{u=1}^N x_{iu}^2 x_{ju}^2 . \quad (2.15)$$

The moment matrix for an experimental design is defined as $N^{-1}(X'X)$. For example, a second order design in two variables ($k=2$) has moment matrix

$$N^{-1}(X'X) = \begin{bmatrix} 1 & [1] & [2] & [11] & [22] & [12] \\ & [11] & [12] & [111] & [122] & [112] \\ & & [22] & [112] & [222] & [122] \\ & & & [1111] & [1122] & [1112] \\ & & & & [2222] & [1222] \\ & & & & & [1122] \end{bmatrix} \quad (2.16)$$

(symmetric)

The inverse of the moment matrix, that is $N(X'X)^{-1}$, is called the precision matrix. The elements of the precision matrix represent the variances and covariances associated with the estimators of the model parameters produced by the design. For a second order experimental design in two variables this is

$$N(X'X)^{-1} = \frac{N}{\sigma^2} \begin{bmatrix} \text{Var}(b_0) & \text{Cov}(b_0 b_1) & \text{Cov}(b_0 b_2) & \text{Cov}(b_0 b_{11}) & \text{Cov}(b_0 b_{22}) & \text{Cov}(b_0 b_{12}) \\ & \text{Var}(b_1) & \text{Cov}(b_1 b_2) & \text{Cov}(b_1 b_{11}) & \text{Cov}(b_1 b_{22}) & \text{Cov}(b_1 b_{12}) \\ & & \text{Var}(b_2) & \text{Cov}(b_2 b_{11}) & \text{Cov}(b_2 b_{22}) & \text{Cov}(b_2 b_{12}) \\ & & & \text{Var}(b_{11}) & \text{Cov}(b_{11} b_{22}) & \text{Cov}(b_{11} b_{12}) \\ & & & & \text{Var}(b_{22}) & \text{Cov}(b_{22} b_{12}) \\ & & & & & \text{Var}(b_{12}) \end{bmatrix} \quad (2.17)$$

(symmetric)

The precision matrix and design moments are important concepts in the development of design criteria.

2.3.2 Rotatability

Box and Hunter (3) suggested that a desirable characteristic for a response surface design is that $\text{Var}(\hat{y})$, the variance of the fitted response surface, be a function of distance from the design center alone and not of direction from the design center. That is, the design should produce variance contours which are hyperspheres, and hence the variance of the fitted surface is unaffected by the orientation of the design about any given design center. Box and Hunter called this design characteristic rotatability, and developed a necessary and sufficient condition for a rotatable design.

A design of order d is rotatable if and only if for all moments of order through $2d$

$$[1^{\alpha_1} 2^{\alpha_2} \dots k^{\alpha_k}] = \frac{\lambda_{\alpha} \prod_{i=1}^k (\alpha_i)!}{2^{\alpha/2} \prod_{i=1}^k \left(\frac{\alpha_i}{2}\right)!} \quad (2.18)$$

if all α_i are even, and

$$[1^{\alpha_1} 2^{\alpha_2} \dots k^{\alpha_k}] = 0, \quad (2.19)$$

if any α_i is odd, where $\alpha = \sum_{i=1}^k \alpha_i$ is the order of the moment.

For a second order design this condition implies that

$$[11] = [22] = \frac{\lambda_2(2!0!)}{2(1!0!)} = \lambda_2, \quad (2.20)$$

$$[1111] = [2222] = \frac{\lambda_4(4!0!)}{2^2(2!0!)} = 3\lambda_4, \quad (2.21)$$

$$[1122] = [2211] = \frac{\lambda_2(2!2!)}{2^2(1!1!)} = \lambda_4, \quad (2.22)$$

and all other moments (i.e., odd moments) are zero.

The moment matrix for a rotatable second order design, therefore, must be of the form

$$N^{-1}(X'X) = \begin{array}{c} \begin{array}{cccccccccccc} & 0 & x_1 & x_2 & \dots & x_k & x_1^2 & x_2^2 & \dots & x_k^2 & x_1x_2 & \dots & x_{k-1}x_k \end{array} \\ \begin{array}{l} 0 \\ x_1 \\ x_2 \\ \vdots \\ x_k \\ x_1^2 \\ x_2^2 \\ \vdots \\ x_k^2 \\ x_1x_2 \\ \vdots \\ x_{k-1}x_k \end{array} \end{array} \begin{bmatrix} 1 & 0 & 0 & \dots & 0 & \lambda_2 & \lambda_2 & \dots & \lambda_2 & 0 & \dots & 0 \\ \hline 0 & \lambda_2 & & & & & & & & & & \\ \hline 0 & & \lambda_2 & 0 & & & & & & & 0 & \\ \hline \vdots & & & \ddots & & & 0 & & & & 0 & \\ \hline 0 & 0 & & & \lambda_2 & & & & & & & \\ \hline \lambda_2 & & & & & 3\lambda_4 & & & & & & \\ \hline \lambda_2 & & & & & & 3\lambda_4 & 0 & & & & 0 \\ \hline \vdots & & 0 & & & & & \ddots & & & & \\ \hline \lambda_2 & & & & & 0 & & & 3\lambda_4 & & & \\ \hline 0 & & & & & & & & & \lambda_4 & & 0 \\ \hline \vdots & & 0 & & & & 0 & & & & \ddots & \\ \hline 0 & & & & & & & & & & 0 & \lambda_4 \end{bmatrix}$$

It should be noted at this point that λ_2 is just a "nuisance" parameter altered by the choice of scaling convention. In fact, when a value is selected for λ_2 , all designs which have that same λ_2 value are said to be scaled to the same convention. The "usual" scaling convention is $\lambda_2 = 1$.

2.3.3 Other Design Criteria

Consider now the precision matrix. Box and Hunter showed that, for a second order rotatable design under the usual scaling convention, the elements of the precision matrix are

$$\frac{N}{\sigma^2} \text{Var}(b_0) = 2\lambda_4^2 (k+2)A \quad (2.23)$$

$$\frac{N}{\sigma^2} \text{Var}(b_i) = 1 \quad (2.24)$$

$$\frac{N}{\sigma^2} \text{Var}(b_{ii}) = [(k+1)\lambda_4 - (k-1)]A \quad (2.25)$$

$$\frac{N}{\sigma^2} \text{Var}(b_{ij}) = \lambda_4^{-1} \quad (2.26)$$

$$\frac{N}{\sigma^2} \text{Cov}(b_0, b_{ii}) = -2\lambda_4 A \quad (2.27)$$

and

$$\frac{N}{\sigma^2} \text{Cov}(b_{ii}, b_{ij}) = (1 - \lambda_4)A, \quad (2.28)$$

where

$$A = [2\lambda_4\{(k+2)\lambda_4 - k\}]^{-1},$$

and all other covariances are zero.

It is readily observable that $\lambda_4 = 1$ will make $\frac{N}{\sigma^2} \text{Cov}(b_{ii}, b_{jj}) = 0$ and hence give an orthogonal rotatable design. One can also note that, if $\lambda_4 = k/k+2$, then $A = \infty$, and the variances are undefined. This implies a singular $(X'X)$ matrix and hence a design which cannot be used.

Clearly the variances are functions of λ_4 . Consider the variance of the fitted response surface, say

$$\begin{aligned} \frac{N}{\sigma^2} \text{Var}(\hat{y}) = \frac{N}{\sigma^2} & \left[\text{Var}(b_0) + \sum_{i=1}^k x_i^2 \text{Var}(b_i) + \sum_{i=1}^k x_i^4 \text{Var}(b_{ii}) \right. \\ & \left. + \sum_{i < j} \sum x_i^2 x_j^2 \text{Var}(b_{ij}) + 2 \sum_{i=1}^k x_i^2 \text{Cov}(b_0, b_{ii}) + 2 \sum_{i < j} \sum x_i^2 x_j^2 \text{Cov}(b_{ii}, b_{jj}) \right] \end{aligned} \quad (2.29)$$

Substituting equations (2.23) through (2.28) into equation (2.29) and simplifying yields

$$\frac{N}{\sigma^2} \text{Var}(\hat{y}) = A \{ 2\lambda_4^2 (k+2) + 2\rho^2 \lambda_4 (\lambda_4 - 1) (k+2) + \rho^4 [(k+1)\lambda_4 - (k-1)] \}$$

where

$$\rho^2 = \frac{\sum_{i=1}^k x_i^2}{N}$$

$\frac{N}{\sigma^2} \text{Var}(\hat{y})$ can now be plotted as a function of ρ for various values of λ_4 . Such a graph can be found in Myers (8).

Box and Hunter observed that large values of λ_4 tend to admit heavy biasing if the fitted model is of a lower order than the true response surface, while small values of λ_4 tend to give poor results at $\rho = 0$ and cause $\frac{N}{\sigma^2} \text{Var}(\hat{y})$ to behave poorly for $\rho > 0$. They suggested that λ_4 should be selected so as to make $\frac{N}{\sigma^2} \text{Var}(\hat{y})$ at the design center ($\rho = 0$) equal to $\frac{N}{\sigma^2} \text{Var}(\hat{y})$ at $\rho = 1$. Designs which satisfy this condition are called uniform precision designs. Values of λ_4 which produce second order uniform precision designs for various numbers of factors (k) are given in Table 1.

Table 1. λ_4 Values for Second Order Rotatable Uniform Precision Designs

k	2	3	4	5	6	7
λ_4	0.7844	0.8385	0.8704	0.8918	0.9070	0.9184

The design criteria presented so far have been variance oriented. That is, rotatability is based entirely on variance, and uniform precision, while acknowledging bias effects by avoiding large values of λ_4 , is primarily concerned with variance.

Box and Draper (1) discussed the importance of considering bias, that is, the errors in a fitted model due to the true surface being of a higher order than the model. They proposed a criterion which considers both variance and bias. Their criterion is based on integrated mean square error.

Consider the equation

$$J = \frac{NK}{\sigma^2} \int_R E[\hat{y} - g]^2 \underline{dx} \quad (2.30)$$

where \hat{y} represents the fitted surface of order d_1 , g represents the true surface of order d_2 ($d_2 > d_1$), R is the region of interest, and $K = \left[\int_R d\underline{x} \right]^{-1}$. Clearly this represents the mean square error between \hat{y} and g integrated over the region of interest R . Expanding the squared portion of equation (2.30) and taking the expectation of each term gives

$$J = \frac{NK}{\sigma^2} \int_R \left[E(\hat{y}^2) - 2gE(\hat{y}) + g^2 \right] dx. \quad (2.31)$$

It is well known (see Myers (8)) that equation (2.31) can be written as

$$J = \frac{NK}{\sigma^2} \int_R \left\{ [E(\hat{y}^2) - E(\hat{y})^2] + [E(\hat{y})^2 - 2gE(\hat{y}) + g^2] \right\} d\mathbf{x} \quad (2.32)$$

or $J = V + B$, (2.33)

where

$$V = \frac{NK}{\sigma^2} \int_R [E(\hat{y}^2) - E(\hat{y})^2] d\mathbf{x}$$

is the integrated variance of \hat{y} , and

$$B = \frac{NK}{\sigma^2} \int_R [E(\hat{y}) - g]^2 d\mathbf{x}$$

is the integrated squared bias.

Unfortunately J cannot be directly minimized without knowledge of the coefficients missing from the fitted model. The approach usually followed is to minimize either V or B . Box and Draper called designs which minimize V "all variance" designs and designs which minimize B "all bias" designs.

In a subsequent paper, Box and Draper (2) showed that even if the variance term is eight times the value of the bias term the optimal design which minimizes J is very close to the "all bias" design. They also provided a series of curves which describe the optimal design moments from "all bias" to "all variance" for various numbers of factors. For the second order "all bias" design they found the design moments should be $\lambda_2 = (0.515)^2$ and $3\lambda_4/\lambda_2^2 = 1.887$. These design moments will be used later to construct designs which will minimize the effects of bias.

2.4 Second Order Designs

2.4.1 Equiradial Designs

The class of designs which this thesis will investigate consists of polygon designs such that all points are equidistant from the design center.

Consider the design matrix for a general equiradial design, say

$$D = \begin{bmatrix} x_{11} & x_{21} & \cdots & x_{k1} \\ x_{12} & x_{22} & \cdots & x_{k2} \\ \cdots & \cdots & \cdots & \cdots \\ x_{1N} & x_{2N} & \cdots & x_{kN} \end{bmatrix}.$$

Since all points are equidistant from the design center

$$\sum_{i=1}^k x_{iu}^2 = \rho^2,$$

where ρ is the distance of each point from the center, i.e., the radius of the polygon.

Now consider the quantity

$$N^{-1} \sum_{i=1}^k \sum_{u=1}^N x_{iu}^2 = N^{-1} \sum_{u=1}^N \left[\sum_{i=1}^k x_{iu}^2 \right] = N^{-1} \sum_{u=1}^N \rho^2 = \rho^2 \quad (2.34)$$

The first term in equation (2.34) can also be written as

$\sum_{i=1}^k \left[N^{-1} \sum_{u=1}^N x_{iu}^2 \right]$. Noting that the term inside the brackets is the second pure moment $[ii]$, which by the usual scaling convention is equal to one,

we have

$$\rho^2 = \sum_{i=1}^k \left[N^{-1} \sum_{u=1}^N x_{iu}^2 \right] = \sum_{i=1}^k [ii] = \sum_{i=1}^k (1) = k \quad (2.35)$$

Thus, under the usual scaling convention, ρ^2 is equal to k .

Now consider the quantity

$$\begin{aligned} \rho^4 &= N^{-1} \sum_{u=1}^N \left[\sum_{i=1}^k x_{iu}^2 \right]^2 = N^{-1} \sum_{u=1}^N \left[\sum_{i=1}^k x_{iu}^4 + \sum_{\substack{i,j \\ (i \neq j)}} x_{iu}^2 x_{ju}^2 \right] \quad (2.36) \\ &= \sum_{i=1}^k \left[N^{-1} \sum_{u=1}^N x_{iu}^4 \right] + \sum_{\substack{i,j \\ (i \neq j)}} \left[N^{-1} \sum_{u=1}^N x_{iu}^2 x_{ju}^2 \right] \end{aligned}$$

We can see that, in the last line of equation (2.36), the first quantity in brackets is simply the fourth pure design moment $[iiii]$ and the second quantity in brackets is the fourth even mixed moment $[iijj]$. Substituting this into equation (2.36) and completing the summations yields

$$k[iiii] + k(k-1)[iijj] = \rho^4 .$$

Since the designs must be rotatable, we can substitute λ_4 for $[iijj]$ and $3\lambda_4$ for $[iiii]$. Also, by applying the usual scaling convention, we can substitute k^2 for ρ^4 . These substitutions yield

$$3k\lambda_4 + k(k-1)\lambda_4 = k^2 ,$$

or

$$\lambda_4 = \frac{k}{k+2} . \quad (2.37)$$

This value of λ_4 makes $(X'X)$ singular. This situation can be remedied in

either of two ways:

- 1) Combine two or more equiradial sets of different radii about the same center, or
- 2) Augment the equiradial design with one or more center points.

We shall see shortly that method 2) is just a special case of method 1).

Box and Hunter (3) show that, for a combination of s equiradial rotatable sets

$$\lambda_4 = \frac{Nk \sum_{w=1}^s n_w \rho_w^4}{(k+2) \left(\sum_{w=1}^s n_w \rho_w^2 \right)^2}, \quad (2.38)$$

where n_w is the number of points in the w^{th} set, ρ_w is the radius of the w^{th} set, and $N = \sum_w n_w$. Now the case of adding center points to an equiradial set is merely the case of $s = 2$, $\rho_2 = 0$. Substituting these values into equation (2.38) provides an expression for λ_4 when there are n_1 points in the equiradial set and n_2 points at the center,

$$\lambda_4 = \frac{k(n_1 + n_2)}{(k+2) n_1}. \quad (2.39)$$

For $k=2$, $\lambda_4 = (n_1 + n_2)/2n_1$. Clearly in this case setting the number of center points equal to the number of radial points will make an orthogonal design. The values of n_2 that make uniform precision (or near uniform precision) and near minimum bias designs for $k=2$ are given in Table 2.

Note that the n_2 values for near minimum bias are based upon a second pure moment of $[ii] = (.515)^2$, while those for uniform precision are based upon the usual scaling convention.

Table 2. Values of n_2 for Uniform Precision and Minimum Bias Equiradial Designs, $k=2$

n_1	5	6	7	8	9	10
n_2 (up)	3	3	4	4	5	6
n_2 (mb)	1	2	2	2	2	3

Now consider the concentric combination of two equiradial sets with $\rho_2 \neq 0$. Equation (2.38) becomes

$$\lambda_4 = \frac{N(n_1 \rho_1^4 + n_2 \rho_2^4)}{2(n_1 \rho_1^2 + n_2 \rho_2^2)^2} \quad (2.40)$$

Let

$$P = \frac{\rho_2}{\rho_1}$$

Then

$$\rho_2 = P\rho_1 \quad (2.41)$$

Substituting equation (2.41) into equation (2.40) yields

$$\lambda_4 = \frac{N(n_1 \rho_1^4 + n_2 P^4 \rho_1^4)}{2(n_1^2 \rho_1^4 + n_2^2 P^2 \rho_1^4 + 2n_1 n_2 \rho_1^4)}$$

Simplifying this equation and collecting terms we have

$$\left[\lambda_4 n_2^2 - \frac{(n_1 + n_2)}{2} n_2 \right] P^4 + 2n_1 n_2 \lambda_4 P^2 + \left[\lambda_4 n_1^2 - \frac{(n_1 + n_2)}{2} n_1 \right] = 0 \quad (2.42)$$

For any combination of n_1 and n_2 , we can substitute the desired values of λ_4 into this equation and solve for the appropriate value of P . It should be noted that, for any particular values of n_1 and λ_4 , the value of n_2 which can be combined concentrically has a lower bound. This lower bound is equal to the number of center points which produced the same value of λ_4 when added to an equiradial set of n_1 points. Any value of n_2 less than the lower bound would require an imaginary value for P , hence the design could not be constructed. For the case of orthogonal combinations where $\lambda_4=1$, this means that we can never have $n_2 < n_1$, i.e., an orthogonal combination design with a radial octagon ($n_1=8$) can never be constructed with an interior set of less than eight points. In the uniform precision case ($\lambda_4 = .7844$) for example, combination designs with radial decagon ($n_1=10$) can never be constructed with an interior set of less than six points. For the minimum bias case λ_4 is sufficiently small that there is not a lower bound on the size of the interior equiradial set.

Table 3 gives the values of P which make various concentric combinations result in orthogonal, uniform precision, and minimum bias designs. Note that again the P values for minimum bias designs are based on a second pure moment of $[ii] = (.515)^2$, while those for orthogonal and uniform precision designs are based on the usual scaling convention of $[ii] = 1$.

2.4.2 Central Composite Designs

We shall compare various equiradial designs against the class of central composite designs. Box and Wilson (4) proposed this class of designs which consists of a 2^k factorial (first order) design of $F = 2^k$

Table 3. Values of $P = \rho_2/\rho_1$ for Orthogonal, Uniform Precision, and Minimum Bias Equiradial Designs, $k=2$

n_1	n_2	$R_{(orth)}$	$R_{(unif\ prec)}$	$R_{(min\ bias)}$
5	5	.000	.374	.581
	6	.204	.413	.597
	7	.267	.438	.607
	8	.304	.455	.614
	9	.330	.467	.618
	10	.348	.475	.620
6	5	----	.321	.560
	6	.000	.374	.581
	7	.189	.408	.595
	8	.250	.431	.604
	9	.287	.447	.611
	10	.314	.459	.615
7	5	----	.256	.537
	6	----	.330	.563
	7	.000	.374	.581
	8	.176	.404	.593
	9	.235	.425	.602
	10	.273	.441	.608
8	5	----	.170	.513
	6	----	.279	.545
	7	----	.337	.566
	8	.000	.374	.581
	9	.167	.401	.592
	10	.223	.420	.560
9	5	----	----	.488
	6	----	.217	.525
	7	----	.295	.550
	8	----	.342	.568
	9	.000	.374	.581
	10	.158	.398	.591
10	5	----	----	.462
	6	----	.129	.505
	7	----	.246	.534
	8	----	.306	.554
	9	----	.346	.569
	10	.000	.374	.581

factorial points augmented by n_2 center points and $2k$ axial points to allow fitting of a second order model. The design matrix for a central composite design in two variables would be

$$D = \begin{bmatrix} -1 & & -1 \\ 1 & & -1 \\ -1 & & 1 \\ 1 & & 1 \\ 0 & & 0 \\ & \dots & \\ 0 & & 0 \\ -\alpha & & 0 \\ \alpha & & 0 \\ 0 & & -\alpha \\ 0 & & \alpha \end{bmatrix} .$$

Since the factorial portion will always contain $F = 2^k$ points and the axial portion will always contain k pairs of points such that one factor at a time has the values $+\alpha$ and $-\alpha$ while all other factors are zero, we can see that in general all odd moments will be zero, and

$$N[iiii] = \sum_{u=1}^N x_{iu}^4 = F + 2\alpha^4 \quad (2.43)$$

$$N[iijj] = \sum_{u=1}^N x_{iu}^2 x_{ju}^2 = F . \quad (2.44)$$

To make these designs rotatable we must require that

$$[iiii] = 3[iijj] . \quad (2.45)$$

Substituting equations (2.43) and (2.44) into equation (2.45) and solving for α gives

$$\alpha = (F)^{\frac{1}{4}}, \quad (2.46)$$

the value of α which will insure rotatability.

It can also be shown that, if we multiply each point by a factor of $[N/(F + 2\alpha^2)]^{\frac{1}{2}}$ to scale the design to the usual convention of $[ii] = 1$, then

$$\lambda_4 = N/(F + 4F^{\frac{1}{2}} + 4) . \quad (2.47)$$

Desired values of λ_4 can be achieved by varying the number of center points n_2 and thus varying N .

Values of n_2 and α for the central composite designs which we will use in this study are given in Table 4 below.

Table 4. Values of n_2 and α for Orthogonal, Uniform Precision, and Minimum Bias Central Composite Designs, $k=2$

	n_2	α
Orthogonal	8	1.414
Uniform Precision	5	1.414
Minimum Bias	2	0.799

CHAPTER III

EXPERIMENTAL DEVELOPMENT

3.1 Background

In Evans' (6) masters thesis and a subsequent paper by Montgomery and Evans (7) the basic work of Brooks (5) was taken as a starting point for the study of second order response surface designs as applied to digital simulation. To the four surfaces of Brooks, Evans added two more. One was a modification of Rosenbrock's curved valley, and the other was a very irregular surface derived from an inverse polynomial function. Evans fitted each of the designs in his study at several points on each surface and at various values of the experimental error variance. He then compared the performance characteristics of the designs.

An investigation similar to that of Evans will be conducted to study further the class of equiradial designs.

3.2 The Response Surfaces

The six response surfaces used by Evans, which will also be used in this study, are shown in Figure 1. Surfaces 1 through 4 in Figure 1 were employed by Brooks, and each has a single maximum $y=1$ at $x_1=1$, $x_2=1$. Figure 1 shows the contour lines $y=0.25$, 0.50 , and 0.75 for these four surfaces. Surface 5 is Evans' modification of Rosenbrock's curved valley. This surface has a maximum of $y=-1$ at $x_1=1$, $x_2=1$. Figure 1(e) shows the contour lines $y=-9$, -5 , -2 , and -1.5 for Surface 5. Surface 6

is Evans' inverse polynomial surface. This surface has a maximum of $y=4.173749909$ at $x_1=2.4475$, $x_2=3.8875$. Figure 1(f) shows the contour lines $y=4.15$, 4.05 , 3.85 , and 3.75 for Surface 6. Mathematically, the surfaces are defined as follows:

Surface 1

$$y = (0.5 + 0.5 x_1)^4 x_2^4 \exp[2 - (0.5 + 0.5 x_1)^4 - x_2^4] + \epsilon .$$

This is a response surface in which the two factors are independent of each other.

Surface 2

$$y = (0.3 + 0.4 x_1 + 0.3 x_2)^4 (0.8 - 0.6 x_1 + 0.8 x_2)^4 \exp[2 - (0.3 + 0.4 x_1 + 0.3 x_2)^4 - (0.8 - 0.6 x_1 + 0.8 x_2)^4] + \epsilon .$$

This is equivalent to Surface 1 rotated approximately 37 degrees, representing a dependence between x_1 and x_2 .

Surface 3

$$y = x_1^2 \exp[1 - x_1^2 - 20.25 (x_1 - x_2)^2] + \epsilon .$$

This surface is a sharp narrow ridge with large flat areas of low response.

Surface 4

$$y = (0.3 x_1^2 + 0.7 x_2^2)^3 \exp[1 - 0.6(x_1 x_2)^2 - (0.3 x_1^2 + 0.7 x_2^2)^3] + \epsilon .$$

This surface is a relatively flat curvilinear ridge.

Surface 5

$$y = - [1 + 100 (x_2 - x_1^2)^2 + (1 - x_1)^2] + \epsilon .$$

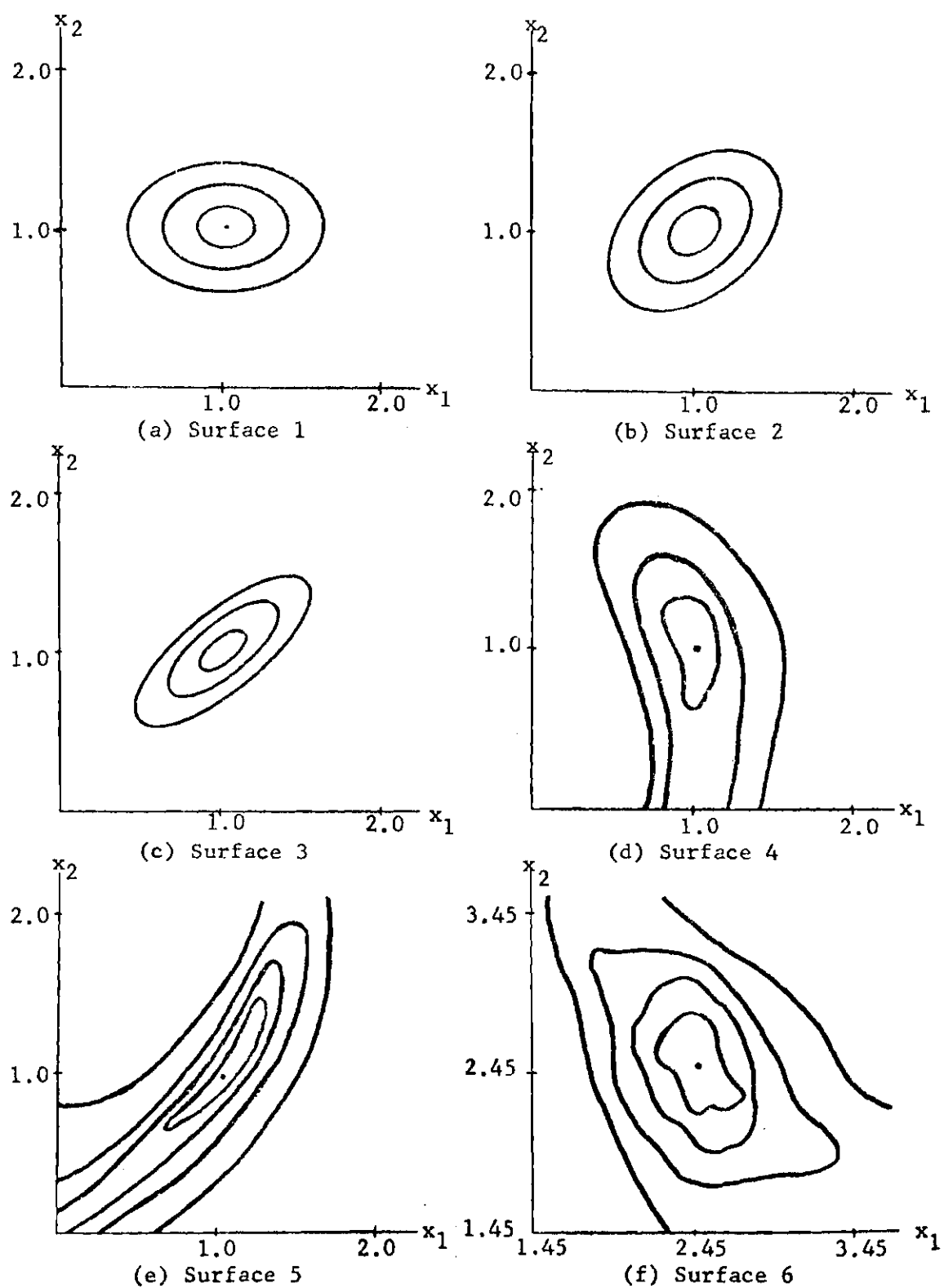


Figure 1. Response Surfaces

This surface is a steep curved ridge.

Surface 6

$$y = [(x_1 x_2) / (.9 + .066 x_1 - .001 x_2 - .01 x_1^2 + .03 x_2^2 + .005 x_1 x_2 + .01 x_1^2 x_2 - .017 x_1 x_2^2 + .013 x_1^2 x_2^2)] + \epsilon .$$

This is a low, bumpy, irregular surface.

3.3 Comparison Between Designs

In order to make valid comparisons between experimental designs and their performance on a group of surfaces, it is necessary that the "spread" of the designs be equal.

The usual measure of spread is the second pure design moment $[ii] = \lambda_2$. The problem is to insure that designs to be compared are scaled to the same convention. As mentioned previously, the usual scaling convention is $\lambda_2 = 1$. In this study, however, we wish to compare a group which includes minimum bias designs. As noted earlier, the minimum bias design for two factors should have $\lambda_2 = (.515)^2$. Therefore, throughout this investigation we will use $\lambda_2 = (.515)^2$ as our scaling convention.

In the case of central composite designs this can be accomplished by multiplying each design point of a central composite design scaled to the usual convention by a factor of (.515). In the case of equiradial designs it is simpler to scale the design by appropriate selection of the design radius ρ_1 .

The design matrix for a rotatable equiradial design in two variables can be generated by the relationship

$$\begin{aligned} x_{1u} &= \rho_1 \cos[\theta + (2\pi u)/n_1] \\ x_{2u} &= \rho_1 \sin[\theta + (2\pi u)/n_1] \end{aligned} \quad (u=0,1,\dots,n_1-1) \quad (3.1)$$

where θ is the angle that the first design point makes with the x_1 axis, and n_1 is the number of points in the equiradial set. Center points must of course be added to complete the design generated by equation (3.1).

To generate a combination design with n_1 radial points, n_2 inner points, and $\rho_2/\rho_1 = P$ we use the relationship

$$\begin{aligned} x_{1u} &= \rho_1 \cos[\theta + (2\pi u)/n_1] \\ x_{2u} &= \rho_1 \sin[\theta + (2\pi u)/n_1] \end{aligned} \quad (u=0,1,\dots,n_1-1) \quad (3.2)$$

$$\begin{aligned} x_{1(n_1+v)} &= P\rho_1 \cos[\theta + (2\pi v)/n_2] \\ x_{2(n_1+v)} &= P\rho_1 \sin[\theta + (2\pi v)/n_2] \end{aligned} \quad (v=0,1,\dots,n_2-1)$$

It can be shown that the second pure design moment for such a design is

$$[ii] = \lambda_2 = \frac{\rho_1^2 (n_1 + n_2 P^2)}{2(n_1 + n_2)} . \quad (3.3)$$

Solving equation (3.3) for ρ_1 gives

$$\rho_1 = \left[\frac{2N\lambda_2}{n_1 + n_2 P^2} \right]^{\frac{1}{2}} . \quad (3.4)$$

For the case of an equiradial set augmented by n_2 center points, P is zero and equation (3.4) reduces to

$$\rho_1 = \left[\frac{2N\lambda_2}{n_1} \right]^{\frac{1}{2}}. \quad (3.5)$$

We can use equations (3.4) and (3.5) to give us the design radii which will make $\lambda_2 = (.515)^2$ in each of our equiradial designs. Appendix A contains representative examples of the design matrices generated in this manner and their associated moment matrices.

A second point which concerns us in the comparison of designs is the coding of the design on the region of interest, i.e., the area of the surface to be investigated. In order that each design investigated cover the same area of the true response surface, a coding factor of $CF = \rho_1/.29$ for equiradial designs and $CF = \alpha/.29$ for central composite designs is used. This will insure that each design covers an area with a radius of .29 surface units. The coding of the designs to the surfaces is given by the relationship

$$x_i(\text{surface}) = \frac{x_i(\text{design})}{CF} \quad (3.6)$$

Evans (6) conducted a series of steepest ascent experiments on each of the surfaces in Figure 1. He calculated an average linear deviation of the optimum found by steepest ascent from the true optimum. This linear deviation was 0.11 surface units for surfaces 3 and 5, and 0.18 for the remaining surfaces.

In this investigation thirty design centers are selected for each surface. On each surface these design centers are equally spaced along a circle about the true optimum with the radius of the circle equal to the average linear deviation determined by Evans. The coding scheme estab-

lished for this investigation insures that the true optimum will always lie within designs centered as described above.

3.4 The Computer Program

This investigation required the use of seven main programs and one common subroutine. The main programs differ only in the portion of the program which generates the designs. The main programs correspond to the following groupings of designs:

- 1) Equiradial sets augmented by center points
- 2) Orthogonal equiradial combinations
- 3) Uniform precision equiradial combinations
- 4) Minimum bias equiradial combinations
- 5) Orthogonal central composite
- 6) Uniform precision central composite
- 7) Minimum bias central composite

The subroutine contains the response functions for the six surfaces.

Appendix B contains a listing of one main program and the surface subroutine. A generalized flow chart of the computer program appears in Figure 2.

The random error component used in the program is assumed to be normally distributed with zero mean and standard deviation σ_{ϵ} . The error standard deviation takes on values from zero, representing no error in the experimental observations through 0.03, 0.06, 0.09, 0.12, and 0.15.

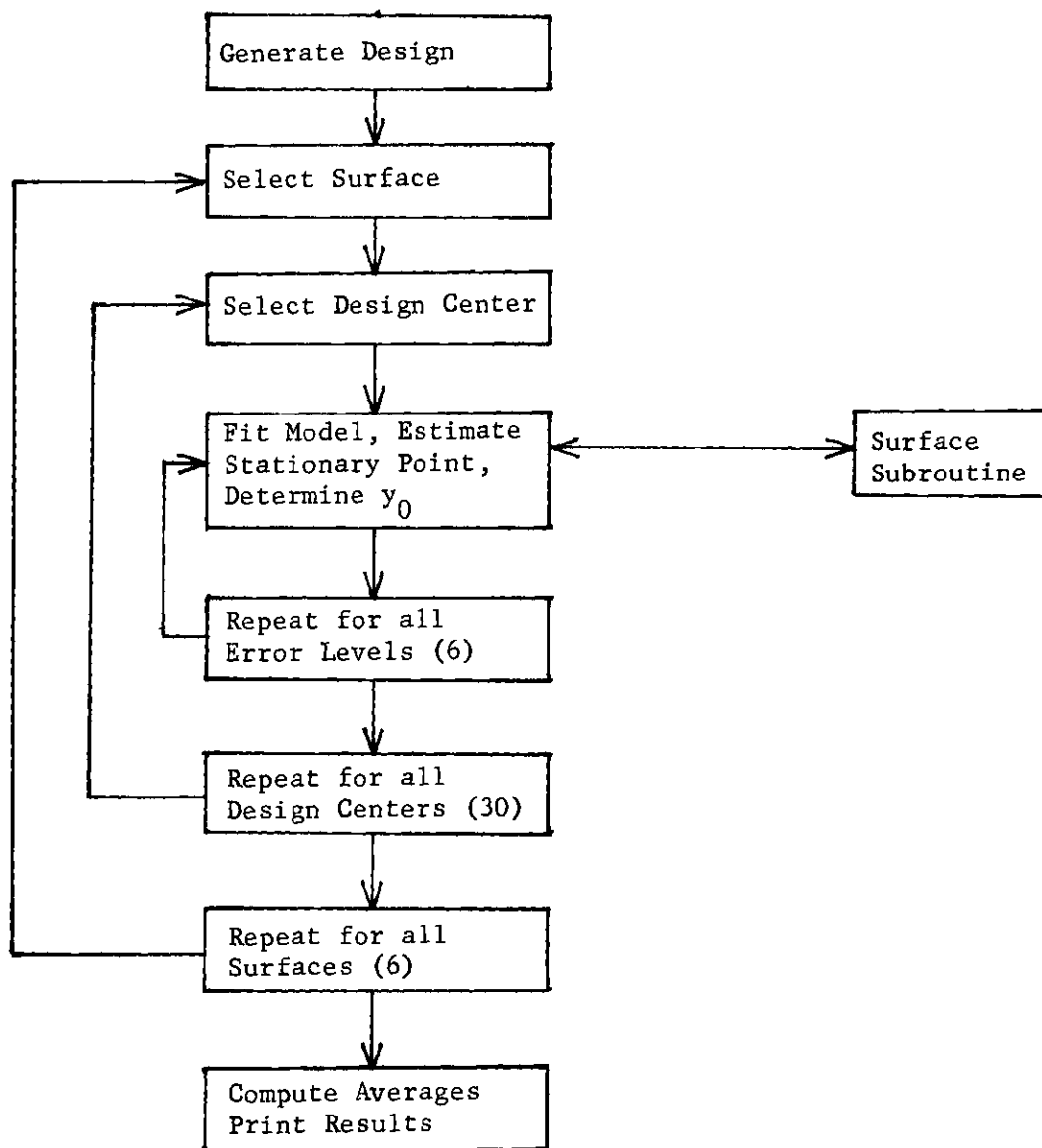


Figure 2. Generalized Flow Chart of Computer Program

CHAPTER IV

EXPERIMENTAL RESULTS

4.1 Measures of Effectiveness

This study applies two measures of effectiveness to the designs investigated. These measures are stationary point deviation (D) and response achievement (R). The first measures the average distance of the estimated optimal factor combination from the true optimal factor combination. Defining (x_{10}, x_{20}) as the estimated optimal combination and (x_{1*}, x_{2*}) as the true optimal combination, the stationary point deviation measure is then

$$D = [(x_{10} - x_{1*})^2 + (x_{20} - x_{2*})^2]^{\frac{1}{2}} . \quad (4.1)$$

The second measure of effectiveness compares the estimated optimal response against the true optimal response. Defining y_0 as the estimated optimum and y_* as the true optimum, the response achievement is then

$$R = \frac{y_0}{y_*} . \quad (4.2)$$

This is simply the percentage of optimal response which is achieved by the estimated response y_0 .

Due to the large number of designs investigated and the fact that the nature of the surface is usually unknown in a response surface investigation, experimental results will be evaluated by design groups only.

A total of 133 designs was investigated. Each design was fitted on each of the six surfaces at 30 different design centers for each surface and six different error levels for each design center. This amounts to 1080 experiments for each design, or a total of 143,640 experiments. The 30 design centers for each surface were preselected to be equally spaced on a circle about the true optimum. The radius of the circle in each case was equal to the average linear deviation of the steepest ascent method as determined by Evans (6). The performance results for each design are averaged over all design centers and tabulated by surface and error level. The individual design performance results may be found in Appendix C.

It is impractical to analyze and compare the individual performance characteristics of 133 designs. Instead, designs will be collected in certain logical groupings, these groups analyzed, and then the "best performers" of each group will be compared. The groupings will be as follows:

- 1) Equiradial sets augmented by center points
- 2) All uniform precision designs
- 3) All orthogonal designs
- 4) All minimum bias designs
- 5) "Best performers" from each of the above.

4.2 Experimental Results by Design Groups

In this section the performance of designs within the various groupings will be analyzed. The results of this analysis should provide insight for the experimenter in the selection of an experimental design

for investigation of an unknown response surface.

4.2.1 Equiradial Sets Augmented by Center Points

This group of designs includes all designs with n_1 ranging from 5 through 10 and n_2 ranging from 1 to n_1 .

Table 5 shows the average distance (D) and response (R) achievements for equiradial pentagon designs with from 1 to 5 center points. The best results among these designs is achieved when $n_0=3$ which corresponds to a uniform precision design. The performance of pentagon designs improves as center points are added up to the uniform precision case and declines thereafter.

Table 6 shows the average D and R achievements for equiradial hexagon designs with from 1 to 6 center points. The best results among these is also achieved when $n_0=3$, the uniform precision configuration. As with the pentagon designs, the hexagon design performance improves with the addition of center points up to uniform precision and then declines.

Table 7 shows the performance achievements for equiradial heptagon designs with from 1 to 7 center points. The best results among the heptagons is achieved when $n_0=1$. Performance of the other designs in this group is irregular and there is no apparent trend associated with the addition of center points.

Table 8 shows performance achievements for equiradial octagon designs with 1 to 8 center points. The best performance in this group is achieved with $n_0=1$. The D achievement is irregular throughout; however, the R achievement declines from $n_0=1$ to $n_0=3$ and improves thereafter.

Table 9 shows performance achievements for equiradial nonegon designs with from 1 to 9 center points. For these designs the best R

Table 5. Distance (D) and Response (R) Achievement for Pentagon Designs

n_1	n_0	σ_ϵ	0.00	0.03	0.06	0.09	0.12	0.15	Avg	Remarks
5	1	D	.20886	.22227	.37046	1.16629	.67986	.35625	.50067	Min Bias
		R	.89848	.90559	.90945	.91161	.91117	.90289	.90653	
	2	D	.20886	.25029	.21209	.27349	.19440	.20438	.22392	
		R	.89848	.90536	.91214	.89831	.90339	.90701	.90412	
	3	D	.20886	.17828	.19105	.19817	.18536	.21288	.19577	Unif Prec
		R	.89848	.90798	.90230	.90146	.91495	.90575	.90500	
	4	D	.20886	.33023	.17538	.20812	.45318	.21397	.26496	
		R	.89848	.90470	.91511	.89801	.89045	.89580	.90043	
	5	D	.20886	.17868	.20773	.19896	.53203	.71468	.34016	Orth
		R	.89848	.90532	.90587	.90928	.87151	.87318	.89394	

Table 6. Distance (D) and Response (R) Achievement for Hexagon Designs

n_1	n_0	σ_e	0.00	0.03	0.06	0.09	0.12	0.15	Avg	Remarks
6	1	D	.35644	.23694	.29346	2.04580	.27965	.25582	.57802	
		R	.91273	.92604	.92502	.92131	.91685	.91226	.91903	
	2	D	.35641	.23676	.32523	.49703	.26240	.26807	.32432	Min Bias
		R	.91273	.92592	.92523	.92181	.91760	.91321	.91942	
	3	D	.35641	.24143	.46032	.25932	.31836	.29018	.32100	Unif Prec
		R	.91273	.92499	.92566	.92346	.92026	.91669	.92063	
	4	D	.35642	.95807	.94970	.50812	.37444	.37640	.58719	
		R	.91273	.88726	.80951	.88992	.89806	.89171	.88153	
	5	D	.35643	.91918	.42379	.37815	2.10942	.36475	.75862	
		R	.91273	.85424	.89816	.89631	.89080	.86883	.88684	
	6	D	.35643	.53235	.32173	.34171	.39284	.79841	.45725	Orth
		R	.91273	.88527	.89725	.89442	.87167	.81816	.87992	

Table 7. Distance (D) and Response (R) Achievement for Heptagon Designs

n_1	n_0	σ_ε	0.00	0.03	0.06	0.09	0.12	0.15	Avg	Remarks
7	1	D	.23230	.45381	.26752	2.30610	.28417	.48981	.67228	
		R	.91268	.92331	.92178	.91768	.91261	.90702	.91585	
	2	D	.23230	.45359	1.99721	2.48976	1.63027	.81217	1.26922	Min Bias
		R	.91268	.91430	.79214	.73536	.79179	.80286	.82485	
	3	D	.23230	.36103	.35481	.37217	.43864	9.31214	1.84518	
		R	.91268	.87610	.87739	.86819	.82209	.75815	.85243	
	4	D	.23230	.39514	.34608	.35044	.73220	1.01151	.51128	Unif Prec
		R	.91268	.87851	.86759	.84384	.78699	.70611	.83262	
	5	D	.23230	.31658	.34969	.63438	.64425	308.65037	51.80459	
		R	.91268	.87993	.86780	.82276	.73962	.68238	.81753	
	6	D	.23230	.52578	.32613	.35713	.48177	8.01068	1.65563	
		R	.91268	.87788	.87952	.87176	.83467	.77874	.85921	
	7	D	.23230	.35285	.33629	.35867	.44116	.92768	.44149	Orth
		R	.91268	.87792	.87597	.86942	.82576	.76523	.85450	

Table 8. Distance (D) and Response (R) Achievement for Octagon Designs

n_1	n_0	σ_ϵ	0.00	0.03	0.06	0.09	0.12	0.15	Avg	Remarks
8	1	D	.22013	.28448	.28739	.32691	.33798	.33776	.29911	
		R	.91756	.91155	.89914	.88250	.86928	.85208	.88869	
	2	D	.22013	.37447	.47941	.84407	1.62640	1.44613	.83177	Min Bias
		R	.91756	.88308	.85836	.73911	.58475	.60835	.76520	
	3	D	.22013	.38947	.48892	1.41168	1.76531	2.02253	1.04967	
		R	.91756	.88319	.85938	.73695	.55534	.60433	.75946	
	4	D	.22013	.37223	.47859	1.19587	1.93390	1.35333	.92568	Unif Prec
		R	.91756	.88303	.85786	.74326	.59756	.60977	.76817	
	5	D	.22013	.35845	.39782	.72275	.57854	1.99065	.71222	
		R	.91756	.88803	.87711	.85221	.80821	.73920	.84705	
	6	D	.22013	.33381	.38385	.82958	.58630	.86900	.53711	
		R	.91756	.88772	.87666	.85200	.80653	.73516	.84594	
	7	D	.22013	.32423	.36416	.48356	1.65411	.61983	.61100	
		R	.91756	.89154	.87840	.85519	.81951	.76264	.85414	
	8	D	.22013	.40307	.33326	.41561	1.02726	.77859	.52965	Orth
		R	.91756	.89353	.87777	.85816	.82586	.77336	.85771	

Table 9. Distance (D) and Response (R) Achievement for Nonegon Designs

n_1	n_0	σ_ϵ	0.00	0.03	0.06	0.09	0.12	0.15	Avg	Remarks
9	1	D	.21998	.96949	1.00514	3.30275	11.61080	1.45137	3.09326	
		R	.91862	.78742	.66438	.56207	.59403	.59582	.68706	
	2	D	.21997	1.13358	.55306	1.07063	2.57669	1.21563	1.12826	Min Bias
		R	.91862	.81178	.79560	.59917	.55717	.58265	.71083	
	3	D	.21997	.73891	.52418	1.09366	2.39160	1.77100	1.12322	
		R	.91862	.83891	.81444	.65283	.58324	.56708	.72919	
	4	D	.21997	.36483	.56936	.50017	5.44648	1.32133	1.40369	
		R	.91862	.88942	.84901	.81639	.75657	.64456	.81243	
	5	D	.21997	.36337	.51063	.68612	1.99586	2.60746	1.06390	Unif Prec
		R	.91862	.88984	.84829	.82220	.75015	.65938	.81475	
	6	D	.21997	.35221	.54613	.51523	.57182	.64661	.47533	
		R	.91862	.89613	.85938	.82678	.79671	.73618	.83897	
	7	D	.21997	.51438	.76356	.63303	.47165	.97193	.59576	
		R	.91862	.89665	.87200	.83523	.80332	.75741	.84720	
	8	D	.21997	.37261	.78487	.46895	.57838	.79262	.53623	
		R	.91862	.89266	.85528	.82584	.78769	.70633	.83107	
	9	D	.21997	20.45302	.77091	.44755	.54126	1.06284	3.91592	Orth
		R	.91862	.89703	.87286	.83675	.80358	.75828	.84785	

achievement is reached when $n_0=9$, the orthogonal case. The corresponding D achievement, however, is the worst of all the nonegon designs. The best D is achieved when $n_0=6$, and the corresponding R is only slightly lower than the best. Again there is no apparent trend in D achievement; however, R achievement improves throughout except when $n_0=8$.

Table 10 shows performance achievements for equiradial decagon designs with from 1 to 10 center points. The best R performance for the decagon designs is achieved by the orthogonal design ($n_0=10$). However, the best D performance is achieved when $n_0=9$, and the R achievement of this design is only slightly less than that of the orthogonal decagon. Among equiradial decagon designs there is no apparent trend associated with the addition of center points.

Considering the entire group of equiradial designs augmented by center points, it can be observed that, when the error level is zero, the addition of more center points to a design has little or no effect on design performance. While this is not true in the presence of error, there is no clear trend for all of these designs as the number of center points is increased. This is shown graphically in Figure 3. The pentagon and hexagon designs perform consistently better than other designs with the same number of center points. The best D achievement was by the uniform precision pentagon, and the best R achievement was by the uniform precision hexagon.

4.2.2 Uniform Precision Designs

Table 11 shows the distance and response achievements for all uniform precision designs. This includes the six uniform precision designs from the previous section which were achieved by an appropriate number of

Table 10. Distance (D) and Response (R) Achievement for Decagon Designs

n_1	n_0	σ_e	0.00	0.03	0.06	0.09	0.12	0.15	Avg	Remarks
10	1	D	.21990	1.20983	.40152	1.80417	1.12934	1.10097	.97762	Min Bias
		R	.91861	.80911	.84063	.66762	.66146	.65664	.77251	
	2	D	.21991	15.13965	.54562	4.11109	1.57115	1.25723	3.80744	
		R	.91861	.88774	.83323	.64500	.66713	.64437	.76601	
	3	D	.21990	.35664	.87498	.41219	.76165	.47474	.51669	
		R	.91861	.90598	.88070	.86891	.83205	.76978	.86267	
	4	D	.21990	.52620	1.39106	.71905	.40357	.77346	.67221	
		R	.91861	.86147	.88188	.85703	.81644	.70113	.83943	
	5	D	.21990	.39817	1.63688	.40017	.39690	.55932	.60189	
		R	.91861	.89741	.88255	.86666	.82617	.74504	.85607	
	6	D	.21990	.38599	1.26147	.40078	.39950	.52937	.53283	
		R	.91861	.90022	.88308	.86659	.82750	.75223	.85804	
	7	D	.21990	4.01245	.72088	.39999	.43615	31.50362	6.21550	
		R	.91861	.83606	.88352	.85284	.79459	.67685	.82708	
	8	D	.21990	.50161	1.68213	.74657	.40177	.73274	.71412	
		R	.91861	.86786	.88214	.85766	.81793	.70678	.84183	
	9	D	.21990	.33295	.65001	.44214	.43186	.44084	.41962	
		R	.91861	.90962	.88390	.87280	.83813	.78689	.86833	
	10	D	.21990	.30356	.60134	.61624	.51594	.42173	.44645	Orth
		R	.91861	.91314	.86859	.87262	.84596	.80943	.87139	

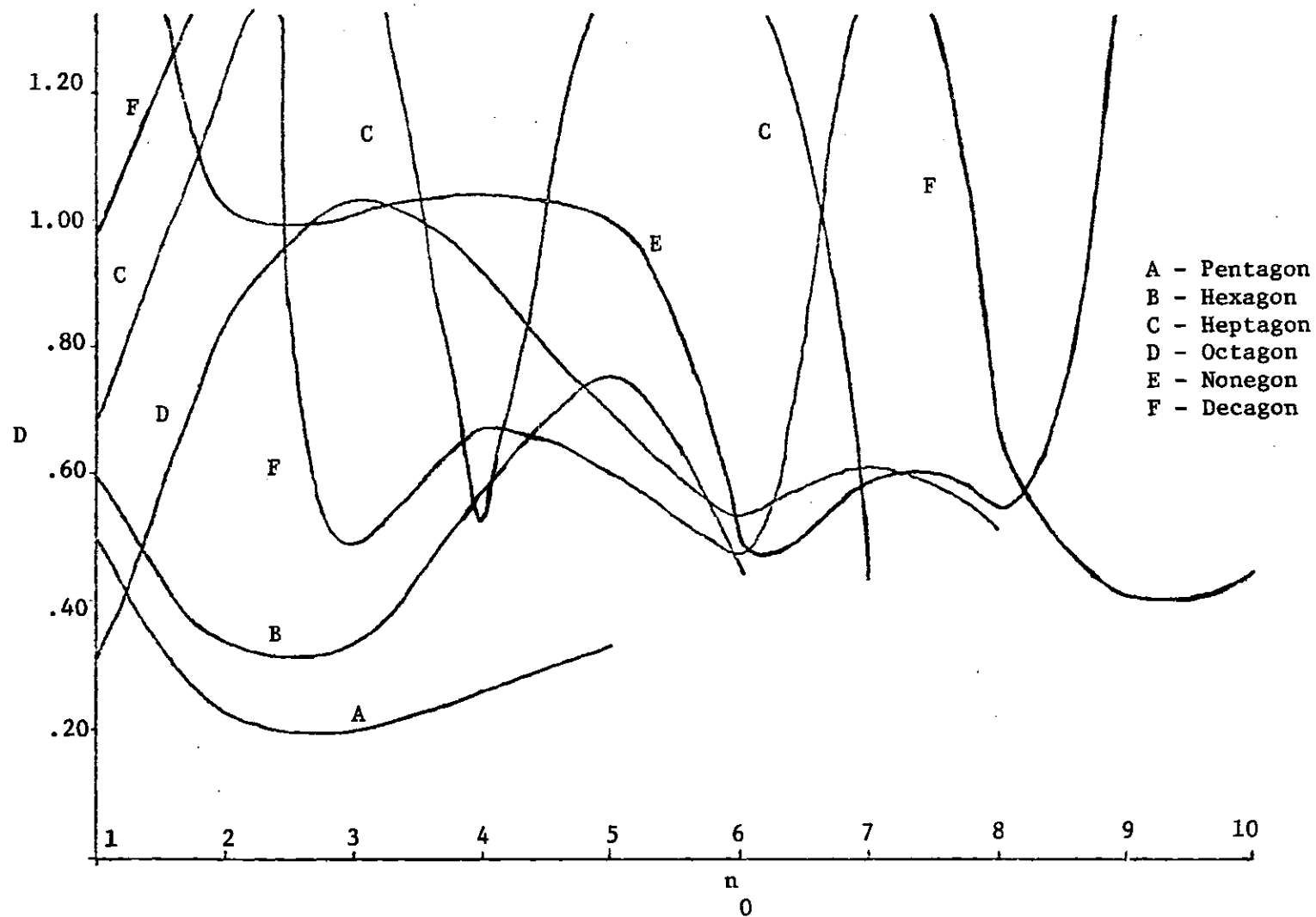


Figure 3. Distance (D) Achievement for Equiradial Designs with Various Center Points n_0

Table 11. Distance (D) and Response (R) Achievement for Uniform Precision Designs

n_1	n_2	ρ_2/ρ_1	σ_ϵ	0.00	0.03	0.06	0.09	0.12	0.15	Avg	Remarks
5	3	.000	D	.20886	.17828	.19105	.19817	.18536	.21288	.19577	Center Points
			R	.89848	.90708	.90230	.90146	.91495	.90575	.90500	
	5	.374	D	.23162	.18474	.20796	.20838	.23741	1.14090	.36850	
			R	.88712	.89794	.89512	.90429	.90079	.85206	.88955	
	6	.413	D	.29792	.21266	.22579	.42690	.89346	.46019	.41949	
			R	.88186	.89206	.89549	.88084	.83311	.85187	.87254	
	7	.438	D	.28724	.21688	.32438	.45984	.77806	.33213	.39976	
			R	.87319	.88082	.88290	.86094	.79984	.84404	.85695	
	8	.455	D	.31281	.20886	.69960	.25850	.34600	.46489	.38178	
			R	.87479	.88179	.88643	.88386	.86696	.84604	.87331	
6	9	.467	D	.41386	.21944	.32102	.29534	.36478	1.13810	.45876	Center Points
			R	.87073	.88081	.88149	.85989	.85784	.85132	.86701	
	10	.475	D	1.62323	.23930	.26526	.35324	.52733	.72058	.62149	
			R	.86426	.87701	.86537	.86143	.85800	.85120	.86288	
	3	.000	D	.35641	.24143	.46032	.25932	.31836	.29018	.32100	
			R	.91273	.92499	.92566	.92346	.92026	.91669	.92063	
	5	.321	D	.27075	.58612	.37350	.72400	.38192	.75159	.51465	
			R	.91850	.87544	.89855	.89492	.88223	.84606	.88595	
	6	.374	D	.23133	.48852	.36184	1.00694	1.12334	2.13162	.89060	
			R	.91780	.86895	.89487	.88759	.85268	.79731	.86986	
	7	.408	D	.22192	.59168	4.61824	.47302	.49456	.52328	1.15378	
			R	.91482	.88658	.81396	.86855	.85769	.82980	.86190	
	8	.431	D	.22442	.32682	4.91450	1.55632	.64692	.66704	1.38934	
			R	.90739	.92083	.84122	.79449	.82329	.82698	.85237	

Table 11. (Continued)

n_1	n_2	ρ_2/ρ_1	σ_ϵ	0.00	0.03	0.06	0.09	0.12	0.15	Avg	Remarks
6	9	.447	D	.24381	.30314	.61057	1.58918	2.30237	1.44793	1.08283	Center Points
			R	.88703	.91866	.91072	.80867	.72550	.74336	.83232	
	10	.459	D	.31634	.35671	.82893	1.27718	.77620	2.12712	.94708	
			R	.87159	.91024	.80227	.80979	.78022	.80962	.83062	
7	4	.000	D	.23230	.39514	.34608	.35044	.73220	1.01151	.51128	
			R	.91268	.87851	.86759	.84384	.78699	.70611	.83262	
	5	.256	D	.23699	.45341	1.28675	.40516	1.07796	.88524	.72425	
			R	.90831	.87120	.85846	.80763	.72979	.67905	.80907	
	6	.330	D	.28328	.50829	5.53907	.59719	.66068	5.01003	2.09976	
			R	.90076	.87666	.87929	.85121	.78807	.72624	.83704	
	7	.374	D	.39567	.64036	.87797	.56750	1.61557	1.27002	.89452	
			R	.89532	.87496	.87478	.85231	.79297	.73307	.83723	
	8	.404	D	.76416	.52214	.43304	.45928	.43673	.67102	.54773	
			R	.89594	.88022	.87409	.85760	.80400	.74757	.84324	
	9	.425	D	.86656	.35727	.40264	.35604	.73476	1.11869	.63933	
			R	.89039	.87368	.87186	.84914	.79865	.74375	.83791	
	10	.441	D	.65562	.39585	.45463	.37186	.76173	.48616	.52098	
			R	.88575	.85492	.86059	.82907	.77001	.72926	.82160	
8	4	.000	D	.22013	.37223	.47859	1.19587	1.93390	1.35333	.92568	Center Points
			R	.91756	.88303	.85786	.74326	.59756	.60977	.76817	
	5	.170	D	.21459	.42348	.34103	.43792	3.89313	.91511	1.03754	
			R	.91949	.88708	.87452	.84742	.79439	.72096	.84064	
	6	.279	D	.21673	.31692	.32211	.37016	.64675	1.28950	.52703	
			R	.92229	.89142	.87193	.84231	.78461	.70487	.83624	

Table 11. (Continued)

n_1	n_2	ρ_2/ρ_1	σ_ϵ	0.00	0.03	0.06	0.09	0.12	0.15	Avg	Remarks
8	7	.337	D	.23657	.28236	.43698	.33179	.41857	.87331	.42993	
			R	.92387	.89572	.87407	.84807	.79887	.73007	.84511	
	8	.374	D	.29242	.29880	.32786	.32759	.41103	.79024	.40799	
			R	.92475	.89613	.87352	.84799	.79648	.72569	.84410	
	9	.401	D	.53802	.47050	.31668	.36555	1.30288	.97865	.66205	
			R	.92519	.88760	.86587	.82915	.76173	.68281	.82539	
	10	.420	D	.92071	.46200	.34623	.37914	.90471	1.63489	.77461	
			R	.92527	.89082	.86632	.83103	.76690	.69579	.82936	
	9	.000	D	.21997	.36337	.51063	.68612	1.99586	2.60746	1.06390	Center Points
			R	.91862	.88984	.84829	.82220	.75015	.65938	.81475	
		.217	D	.21339	.28087	.60043	.48432	.57236	5.22649	1.22964	
			R	.92141	.90439	.86199	.81012	.76854	.68129	.82462	
		.295	D	.21961	.25791	.41394	1.02556	.75877	.70232	.56302	
			R	.92330	.90855	.86277	.81402	.77113	.69678	.82942	
		.342	D	.24083	.25255	.43754	.92009	1.34032	1.66638	.80962	
			R	.92440	.90485	.84381	.77988	.67949	.64422	.79611	
		.374	D	.29347	.24866	.31408	.98289	.64960	.87962	.56139	
			R	.92500	.90838	.85953	.80758	.74742	.64395	.81531	
	10	.398	D	.47961	.25992	.26334	.36882	1.03791	1.12222	.58864	
			R	.92529	.91247	.88635	.83299	.78257	.72045	.84335	
10	6	.000	D	.21990	.38599	1.26147	.40078	.39950	.52937	.53283	Center Points
			R	.91861	.90022	.88308	.86659	.82750	.75223	.85804	
	6	.129	D	.21620	.36801	.62473	.43424	.40632	.53670	.43103	
			R	.91966	.90252	.87988	.86484	.82571	.75091	.85726	

Table 11. (Continued)

n_1	n_2	ρ_2/ρ_1	σ_E	0.00	0.03	0.06	0.09	0.12	0.15	Avg	Remarks
10	7	.246	D	.21389	1.97701	.60013	.58084	1.24926	13.36485	2.99766	
			R	.92209	.85723	.87674	.84700	.77706	.66838	.82475	
	8	.306	D	.22239	.50408	.41929	.76818	.47612	.61363	.49895	
			R	.92357	.86199	.87524	.85634	.80300	.69572	.83598	
	9	.346	D	.24417	.28169	.34419	.44784	.56594	.83026	.45235	
			R	.92448	.91802	.90219	.85382	.83101	.78602	.86926	
	10	.374	D	.29353	.27528	.48957	.35064	.49061	.60934	.41816	
			R	.92501	.92067	.88414	.86579	.82690	.79767	.87003	
--	--	CCD	D	.22006	.25483	.31944	1.21097	.77591	.44144	.53711	CCD
			R	.91756	.91905	.91525	.84334	.79089	.88840	.87908	

center points, all uniform precision equiradial combination designs achieved by an appropriate value of ρ_2/ρ_1 , and the uniform precision ccd.

The best performances in this group are achieved by the same designs which were best in the last group. That is, the uniform precision pentagon has the best distance achievement, and the uniform precision hexagon has the best response achievement. There are no clear trends associated with the number of points in the interior equiradial set. However, it can be observed that, for each combination of equiradial sets, the best performance is generally achieved when the larger set has the smaller radius. For example, it is seen that the pentagon with interior hexagon performs better than the hexagon with interior pentagon. This is shown graphically for pentagon combinations in Figure 4. It can also be seen that the pentagon and hexagon designs perform consistently better than larger designs when the same equiradial sets are used for the interior portion.

These observations are somewhat gratifying since they indicate that better performance and economy in the number of observations can be gained simultaneously. In this respect it should be noted that the uniform precision pentagon and hexagon both performed considerably better than the uniform precision central composite design.

4.2.3. Orthogonal Designs

This section analyzes the performance of all orthogonal designs as shown in Table 12 including the six obtained by augmenting equiradial sets with center points, 15 obtained by appropriate selection of ρ_2/ρ_1 for equiradial combinations, and the orthogonal ccd.

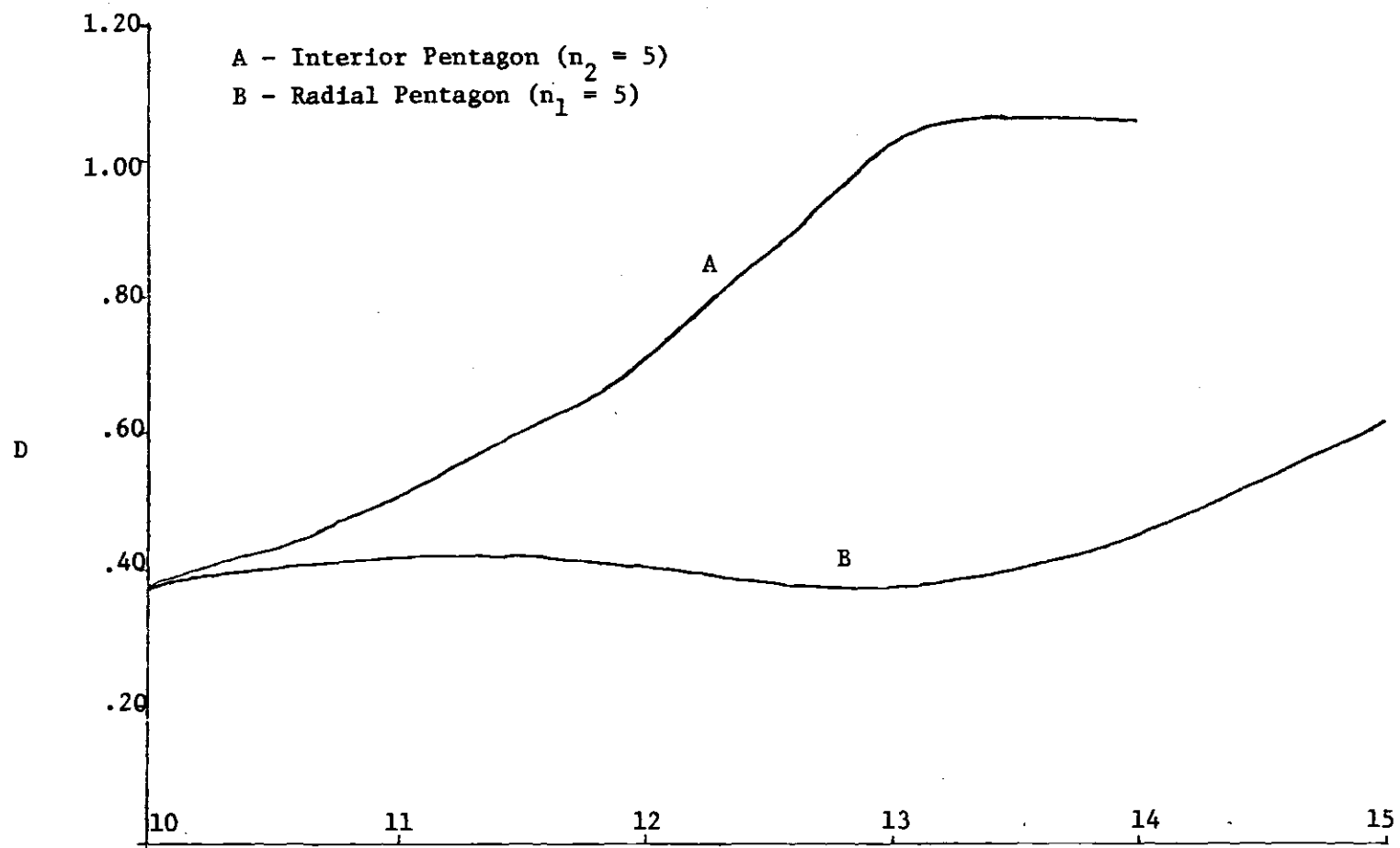


Figure 4. Distance (D) Achievement for Uniform Precision Combination Designs with Radial Versus Interior Pentagons

Table 12. Distance (D) and Response (R) Achievement for Orthogonal Designs

n_1	n_2	ρ_2/ρ_1	σ_ϵ	0.00	0.03	0.06	0.09	0.12	0.15	Avg	Remarks
5	5	.000	D	.20886	.17868	.20773	.19896	.53203	.71468	.34016	Center Points
			R	.89848	.90532	.90587	.90928	.87151	.87318	.89394	
	6	.204	D	.23285	.19876	.22661	.42125	.38805	.26589	.28890	
			R	.89570	.90517	.90128	.88126	.84906	.87737	.88497	
	7	.267	D	.28949	.18804	.35663	.57526	.26260	.26026	.32205	
			R	.88747	.89857	.88215	.82551	.87552	.85759	.87114	
	8	.304	D	.36576	.21146	.21403	.27078	1.18130	1.26082	.58403	
			R	.88474	.89326	.89779	.89066	.84717	.78883	.86707	
	9	.330	D	.42978	.19170	.24993	.23876	.31726	.82822	.37594	
			R	.88470	.89063	.89320	.89006	.87850	.83027	.87789	
6	10	.348	D	.44283	.18870	.20680	.26774	.41592	1.45325	.49587	Center Points
			R	.88198	.89004	.89439	.87415	.85943	.76542	.86090	
	6	.000	D	.35643	.53235	.32173	.34171	.39284	.79841	.45725	
			R	.91273	.88527	.89725	.89442	.87167	.81816	.87992	
	7	.189	D	1.96130	.80166	.43259	.47982	.40180	.49533	.76208	
			R	.91688	.89539	.87875	.89724	.88605	.86045	.88913	
	8	.250	D	.40348	.81806	.45280	.45635	.42682	.51057	.51135	
			R	.91856	.89429	.87781	.89586	.88239	.85425	.88719	
	9	.287	D	.29003	.44034	3.12861	.69551	.44802	.49995	.91708	
			R	.91895	.89255	.81129	.88052	.87607	.85835	.87295	
7	10	.314	D	.25244	.39620	2.20645	1.14886	.41282	.41430	.80518	Center Points
			R	.91857	.91186	.82454	.87967	.87507	.85499	.87745	
7	7	.000	D	.23230	.35285	.33629	.35867	.44116	.92768	.44149	Center Points
			R	.91268	.87792	.87597	.86942	.82576	.76523	.85450	

Table 12. (Continued)

n_1	n_2	ρ_2/ρ_1	σ_ϵ	0.00	0.03	0.06	0.09	0.12	0.15	Avg	Remarks
7	8	.176	D	.22856	.38896	.34222	.40664	.76698	1.00882	.52370	
			R	.91219	.87258	.88132	.86792	.82467	.76372	.85373	
	9	.235	D	.23184	3.27039	.38135	.68129	.59979	.98944	1.02568	
			R	.90945	.87229	.88232	.86762	.82281	.76077	.85254	
	10	.273	D	.24110	.62858	.38719	.64470	.83082	3.94874	1.11352	
			R	.90549	.87195	.87527	.85362	.80256	.73180	.84012	
8	8	.000	D	.22013	.40307	.33326	.41561	1.02726	.77859	.52965	Center Points
			R	.91756	.89353	.87777	.85816	.82587	.77336	.85771	
	9	.167	D	.21223	.44648	.35406	.41321	1.43192	.86797	.62098	
			R	.92055	.88779	.87803	.85284	.80846	.73966	.84789	
	10	.223	D	.20884	.62125	.38582	1.18015	.42472	1.10431	.65418	
			R	.92248	.89427	.88026	.85669	.81371	.75603	.85391	
9	9	.000	D	.21997	20.45302	.77091	.44755	.54126	1.06284	3.91592	Center Points
			R	.91862	.89703	.87286	.83675	.80358	.75828	.84785	
	10	.158	D	.21276	.28469	.42191	.51274	.41589	.43018	.37970	
			R	.92112	.91103	.88379	.85352	.82553	.79356	.86476	
10	10	.000	D	.21990	.30356	.60134	.61624	.51594	.42173	.44645	Center Points
			R	.91861	.91314	.86859	.87262	.84596	.80943	.87139	
--	--	CCD	D	.22006	.26912	.87281	.74201	.76577	.77383	.60727	CCD
			R	.91756	.91014	.83659	.84382	.81905	.75745	.84744	

The best performances in this group are provided by two of the pentagon designs. The center point configuration of the orthogonal pentagon gave the best R achievement, and the pentagon with interior hexagon gave the best D achievement. As in the uniform precision case, both of these designs performed considerably better than the central composite design. It can also be seen again in this group that pentagon and hexagon designs perform generally better than larger designs with the same interior sets, although the distinction is not always as clear in this group as it was among uniform precision designs. This is shown graphically in Figure 5.

4.2.4 Minimum Bias Designs

Table 13 shows the performance achievements for all minimum bias designs, including six center point cases, 36 equiradial combination cases, and the central composite case.

The best performances in this group, as in the other groups, were achieved by pentagon and hexagon designs. The best response achievement was given by the hexagon augmented by two center points. The equiradial combination of a pentagon inside a pentagon gave a slightly better distance achievement, but a much poorer response achievement.

Once again, the pentagon and hexagon consistently out performed larger designs of similar configuration, and the best combination designs were those with $n_2 > n_1$. These results are shown in Figures 6 and 7.

4.2.5 "Best Performers"

Table 14 and Figure 8 show a summary of the best performing designs from each of the preceding groups. Since all of these were equiradial designs, the three central composite designs considered are also included

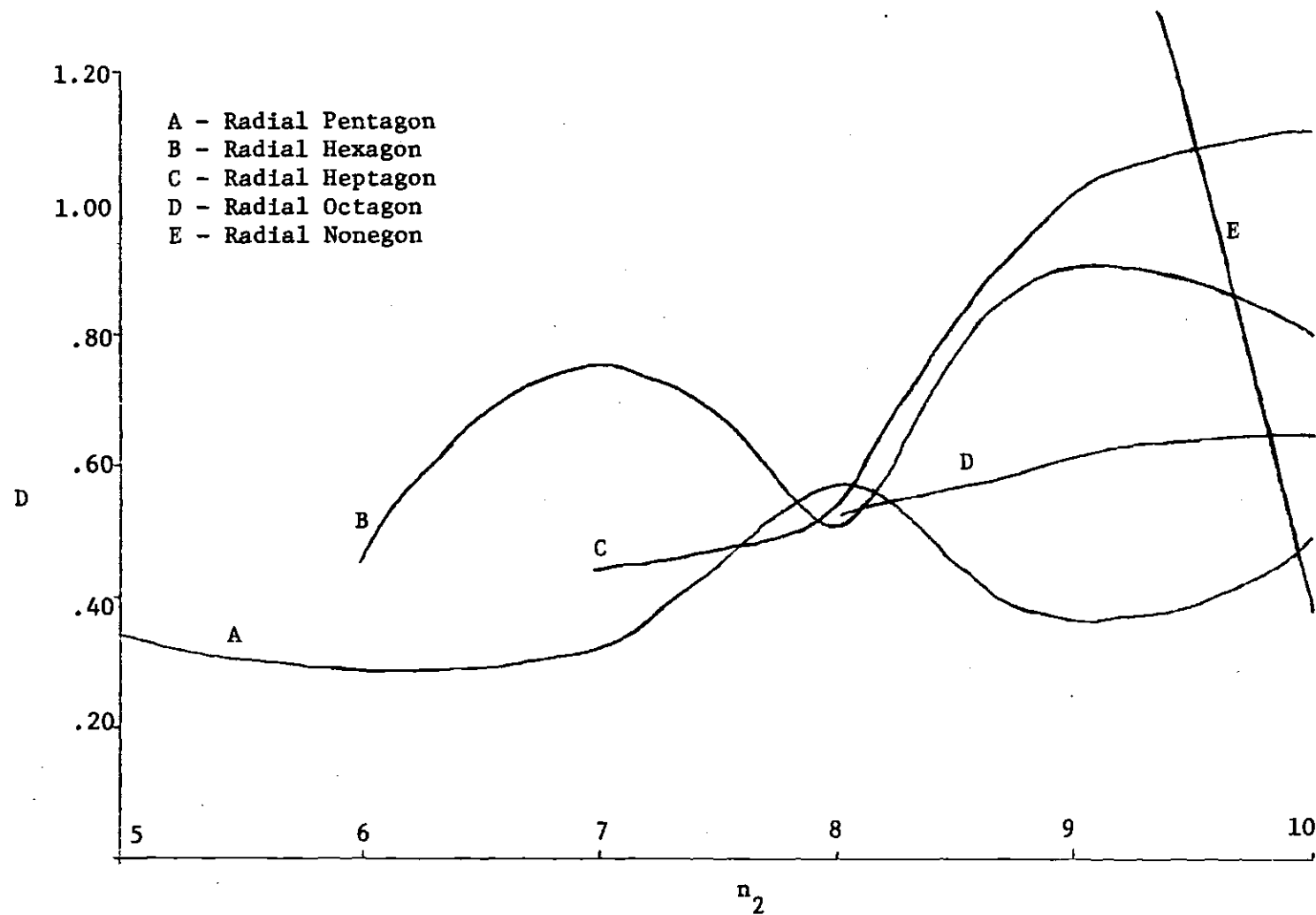


Figure 5. Distance (D) Achievement for Orthogonal Combination Designs with Various Interior Equiradial Sets

Table 13. Distance (D) and Response (R) Achievement for Minimum Bias Designs

n_1	n_2	ρ_2/ρ_1	σ_ϵ	0.00	0.03	0.06	0.09	0.12	0.15	Avg	Remarks
5	1	.000	D	.20886	.22227	.37046	1.16629	.67986	.35625	.50067	Center Points
			R	.89848	.90559	.90945	.91161	.91117	.90289	.90653	
	5	.581	D	.22230	.26293	.30122	.23974	.31482	.48988	.30515	
			R	.86661	.87394	.88466	.89063	.88550	.86369	.87750	
	6	.597	D	.26234	.33121	.34293	1.14951	.56656	.54690	.53324	
			R	.86604	.88040	.88710	.87607	.86864	.86418	.87374	
	7	.607	D	.30938	1.57676	.38433	1.74427	.56598	.71587	.88276	
			R	.86095	.87349	.87996	.87321	.86314	.85620	.86782	
	8	.614	D	.54199	.85541	.83647	.87519	.55989	.44120	.68503	
			R	.85479	.86775	.85822	.85948	.85919	.85944	.85981	
	9	.618	D	1.50289	.86947	5.82923	.54067	.48328	.44565	1.61186	
			R	.84956	.85464	.87418	.89175	.89771	.89723	.87751	
	10	.620	D	.92969	.66572	.56271	.48035	.44725	.48945	.59586	
			R	.84389	.85982	.87215	.88419	.88583	.89071	.87277	
6	2	.000	D	.35641	.23676	.32523	.49703	.26240	.26807	.32432	Center Points
			R	.91273	.92592	.92523	.92181	.91760	.91321	.91942	
	5	.560	D	.30405	1.00728	.62236	.67204	.49711	.43903	.59031	
			R	.87738	.87520	.84372	.86805	.85776	.82444	.85776	
	6	.581	D	1.33722	.63722	.40496	.70504	2.35223	2.01375	1.24174	
			R	.88477	.85024	.85214	.82802	.80113	.76156	.82964	
	7	.595	D	.40585	.47091	.68207	1.72175	4.39966	3.86369	1.92399	
			R	.88886	.89754	.88771	.76229	.68311	.71017	.80495	

Table 13. (Continued)

n_1	n_2	ρ_2/ρ_1	σ_ϵ	0.00	0.03	0.06	0.09	0.12	0.15	Avg	Remarks
6	8	.604	D	.32133	.27511	.75652	.85780	.51527	.42476	.52513	Center Points
			R	.88803	.90193	.90037	.89937	.88663	.87309	.89157	
	9	.611	D	.40232	.29627	.30467	1.75401	.36454	.41385	.58928	
			R	.88861	.90377	.90776	.89671	.89049	.86877	.89269	
	10	.615	D	.24613	.58931	.37594	.44178	.58812	1.29733	.58977	
			R	.89276	.90132	.90182	.86857	.83808	.73427	.85614	
7	2	.000	D	.23230	.45359	1.99721	2.48976	1.63027	.81217	1.26922	
			R	.91268	.91430	.79214	.73536	.79179	.80286	.82485	
	5	.537	D	.30297	.66928	.97826	1.95662	.71570	1.22093	.97396	
			R	.89719	.84850	.82468	.75156	.70302	.66406	.78150	
	6	.563	D	.31861	.32094	.46231	.93393	12.66450	.66424	2.56076	
			R	.89390	.87136	.82368	.78369	.74343	.71835	.80574	
	7	.581	D	1.29916	.36018	1.06864	.40087	.55965	2.46331	1.02530	
			R	.88944	.85526	.83369	.83169	.77301	.73194	.81917	
	8	.593	D	.29354	.36617	3.94349	.58394	.88159	1.43925	1.25133	
			R	.89661	.87723	.82415	.80542	.76816	.73635	.81799	
	9	.602	D	.29905	.41356	.81775	.66108	1.10188	.60088	.64904	
			R	.89690	.86246	.81992	.79793	.75978	.73398	.81183	
8	10	.608	D	.29845	.47551	.67180	1.42092	.60808	1.36779	.80709	Center Points
			R	.89939	.83768	.80619	.77895	.74522	.70578	.79554	
	2	.000	D	.22013	.37447	.47941	.84407	1.62640	1.44613	.83177	
			R	.91756	.88308	.85836	.73911	.58475	.60835	.76520	

Table 13. (Continued)

n_1	n_2	ρ_2/ρ_1	σ_ϵ	0.00	0.03	0.06	0.09	0.12	0.15	Avg	Remarks
8	5	.513	D	.35051	.88788	.57607	.53491	5.03161	1.10313	1.41402	
			R	.90341	.85485	.82036	.75913	.67836	.61948	.77260	
	6	.545	D	.21718	.35564	2.35298	.53966	3.01705	1.39529	1.31297	
			R	.91556	.87005	.82877	.75635	.66654	.58260	.76996	
	7	.566	D	.23198	.29965	.63218	2.13299	.91248	1.44099	.94171	
			R	.91214	.87469	.83005	.77355	.68389	.61271	.78117	
	8	.581	D	.21506	.29318	.41934	.86753	.65479	1.38928	.63986	
			R	.91329	.87600	.82364	.77809	.72905	.68724	.80122	
	9	.592	D	.21163	.40259	.49984	1.09105	1.49494	3.28756	1.16460	
			R	.91938	.85929	.80019	.75480	.68536	.60573	.77079	
	10	.560	D	.21825	.37418	.49340	.90678	.51136	1.47173	.66262	
			R	.91785	.85845	.80731	.75785	.72995	.68737	.79313	
9	2	.000	D	.21997	1.13358	.55306	1.07063	2.57669	1.21563	1.12826	Center Points
			R	.91862	.81178	.79560	.59917	.55717	.58265	.71083	
	5	.488	D	.36324	.29772	.55330	2.14029	4.89869	2.89077	1.85734	
			R	.92369	.88759	.75816	.72179	.61338	.64203	.75777	
	6	.525	D	.21072	1.65128	.60917	1.23479	.68113	4.96055	1.55794	
			R	.92359	.86354	.78003	.73406	.70380	.61092	.76932	
	7	.550	D	.27852	2.43588	.88698	2.34281	.71241	1.26718	1.32063	
			R	.89697	.88334	.80658	.67894	.71062	.65190	.77139	
	8	.568	D	.23208	.36346	.61467	.83290	.89311	15.87416	3.13506	
			R	.90338	.87551	.78837	.70902	.64984	.59744	.75393	
	9	.581	D	.22337	.29783	2.17575	4.97808	.96659	3.93374	2.09573	
			R	.91068	.88074	.81365	.69435	.67728	.62903	.76762	

Table 13. (Continued)

n_1	n_2	ρ_2/ρ_1	σ_ϵ	0.00	0.03	0.06	0.09	0.12	0.15	Avg	Remarks
9	10	.591	D	.24247	.26519	.36012	5.00554	.51163	.56579	1.15845	Center Points
			R	.90435	.89102	.84151	.79078	.74102	.69173	.81007	
10	3	.000	D	.21990	.35664	.87498	.41219	.76165	.47474	.51669	
			R	.91861	.90598	.88070	.86891	.83205	.76979	.86267	
	5	.462	D	.79204	.36074	.36717	.53101	.87756	.55883	.58123	
			R	.92351	.91413	.89496	.80263	.74487	.74449	.83743	
	6	.505	D	.21987	.64853	.45182	1.65271	.38825	.44030	.63358	
			R	.92478	.90669	.87179	.76786	.81865	.78033	.84502	
	7	.534	D	.20363	1.18650	1.23455	.34233	.47679	2.59240	1.00603	
			R	.92404	.81420	.82887	.83949	.74364	.68039	.80510	
	8	.554	D	.32332	3.91808	2.82684	.36637	.36489	.65217	1.40861	
			R	.89324	.82646	.87328	.85564	.81035	.71047	.82824	
	9	.569	D	.25069	.44319	.51534	2.36629	.58099	.37540	.75532	
			R	.90487	.89685	.87323	.85403	.83745	.83489	.86689	
	10	.581	D	.23219	.27974	.90946	.49351	1.02912	.57291	.58616	
			R	.90610	.90190	.85170	.87262	.83405	.82555	.86532	
--	--	CCD	D	.22006	.51976	.37006	1.25218	1.19974	.75740	.71987	CCD
			R	.91757	.87166	.85644	.64454	.58832	.71170	.76504	

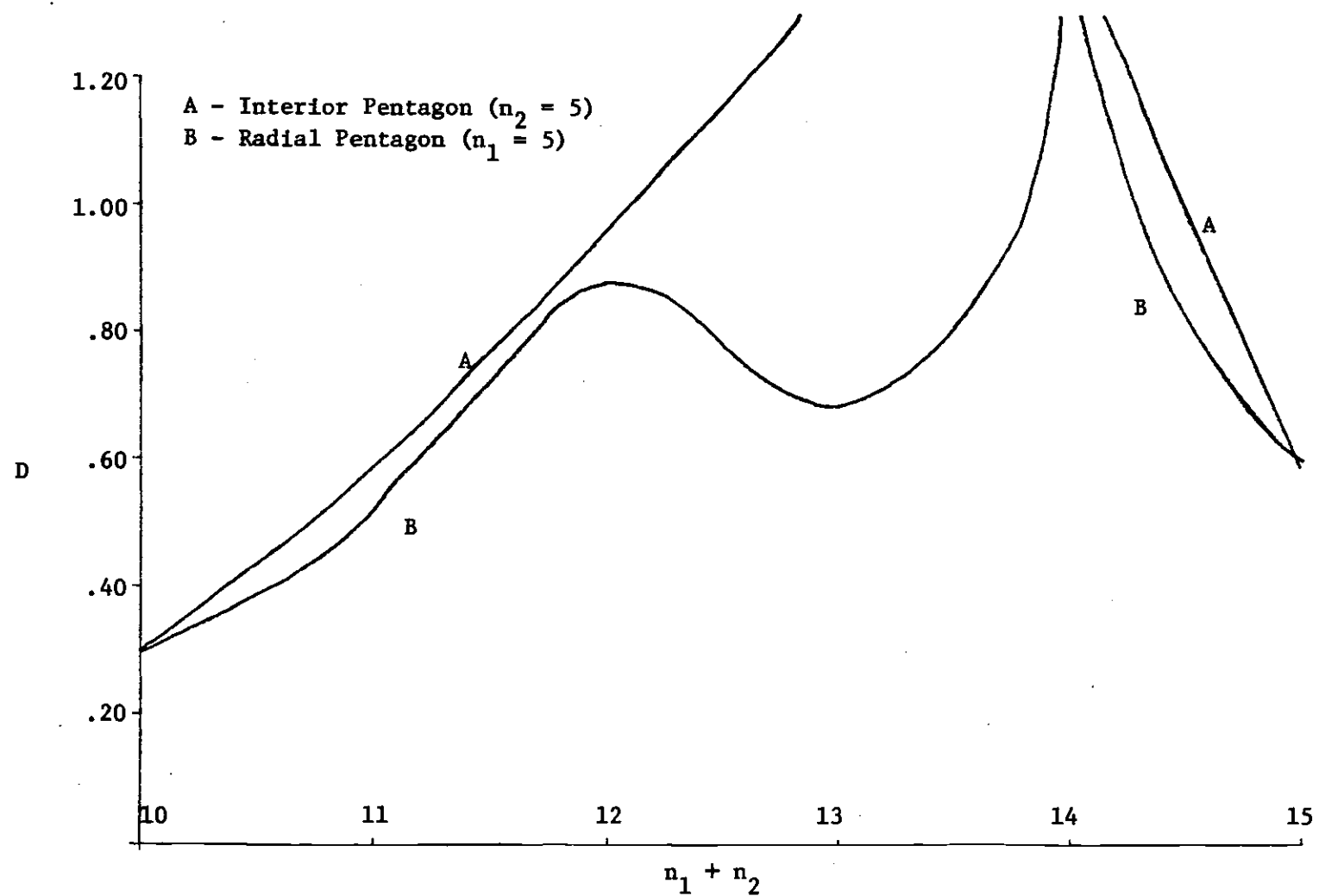


Figure 6. Distance (D) Achievement for Minimum Bias Combination Designs with Radial Versus Interior Pentagons

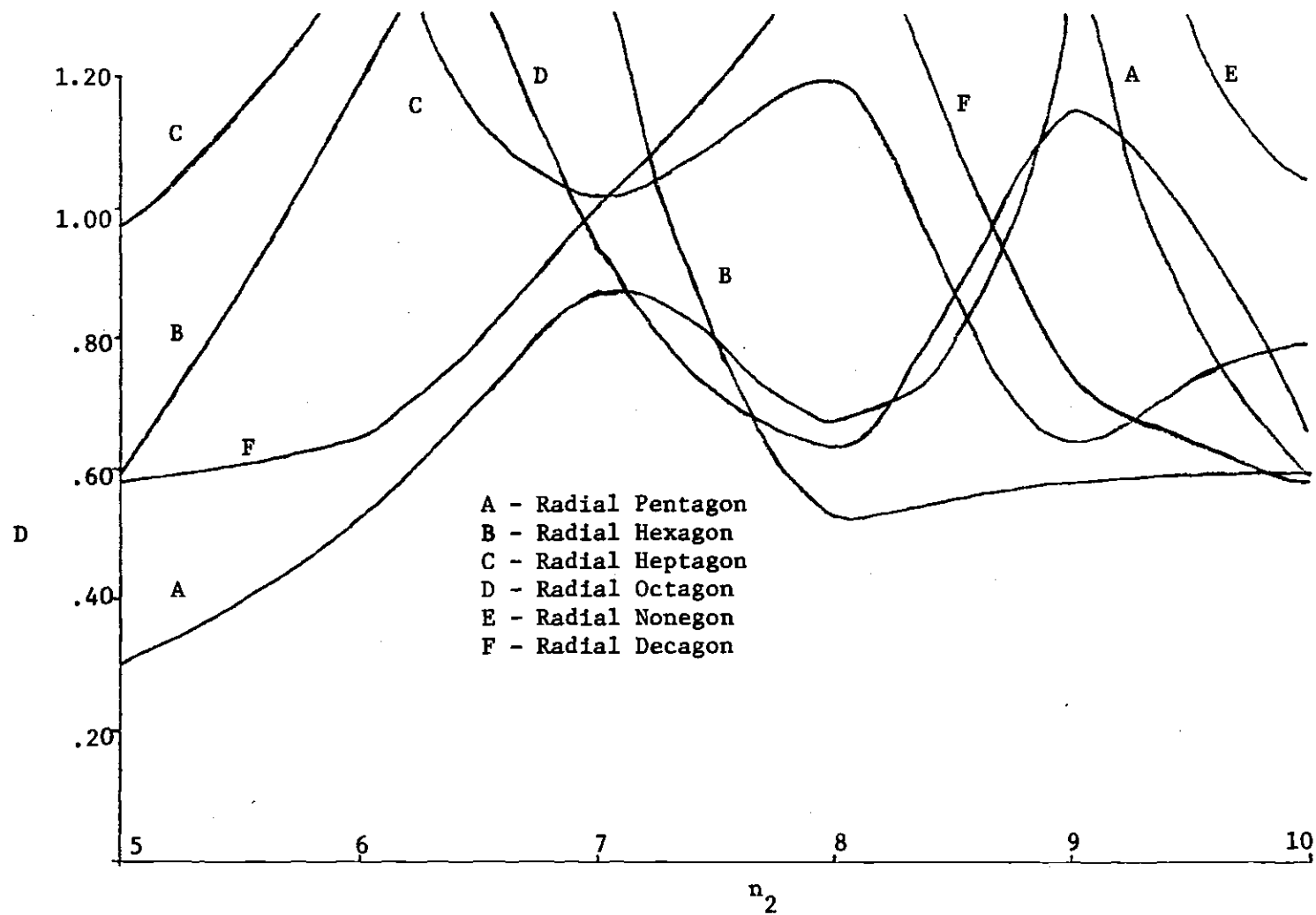


Figure 7. Distance (D) Achievement for Minimum Bias Combination Designs with Various Interior Equiradial Sets

Table 14. Distance (D) and Response (R) Summary for "Best Performers"

n_1	n_2	ρ_2/ρ_1	σ_ϵ	0.00	0.03	0.06	0.09	0.12	0.15	Avg	Remarks
5	3	.000	D	.20886	.17828	.19105	.19817	.18536	.21288	.19577	Unif Prec Center Points
			R	.89848	.90708	.90230	.90146	.91495	.90575	.90500	
6	3	.000	D	.35641	.24143	.46032	.25932	.31836	.29018	.32100	Unif Prec Center Points
			R	.91273	.92499	.92566	.92346	.92026	.91669	.92063	
5	5	.000	D	.20886	.17868	.20773	.19896	.53203	.71468	.34016	Orthogonal Center Points
			R	.89848	.90532	.90587	.90928	.87151	.87318	.89394	
5	6	.204	D	.23285	.19876	.22661	.42125	.38805	.26589	.28890	Orthogonal Combination
			R	.89570	.90517	.90128	.88126	.84906	.87737	.88497	
6	2	.000	D	.35641	.23676	.32523	.49703	.26240	.26807	.32432	Min Bias Center Points
			R	.91273	.92592	.92523	.92181	.91760	.91321	.91942	
5	5	.581	D	.22230	.26293	.30122	.23974	.31482	.48988	.30515	Min Bias Combination
			R	.86661	.87394	.88466	.89063	.88550	.86369	.87750	
--	--	CCD	D	.22006	.25483	.31944	1.21097	.77591	.44144	.53711	Unif Prec CCD
			R	.91756	.91905	.91525	.84334	.79089	.88840	.87908	
--	--	CCD	D	.22006	.26912	.87281	.74201	.76577	.77383	.60727	Orthogonal CCD
			R	.91756	.91014	.83659	.84382	.81905	.75745	.84744	
--	--	CCD	D	.22006	.59176	.37006	1.25218	1.19974	.75740	.71987	Min Bias CCD
			R	.91757	.87166	.85644	.64454	.58832	.71170	.76504	

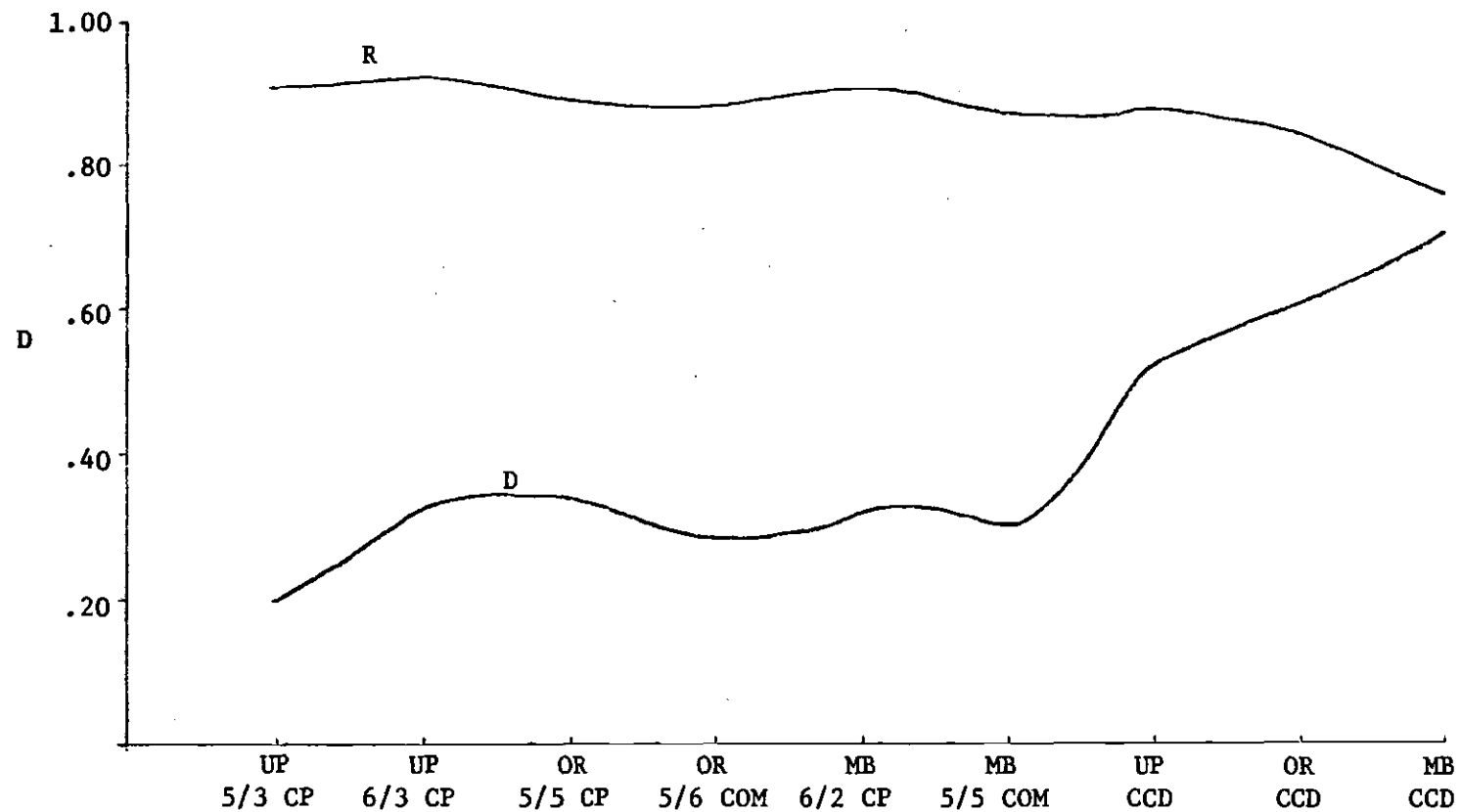


Figure 8. Average Distance (D) and Response (R) Achievement for "Best Performers"

for a final comparison. Clearly the best of this group are the uniform precision pentagon and hexagon. In the case of no experimental error, the performance of all designs in the group is closely similar.

4.3 Discussion

The underlying reasons for several of the observed results are not clearly apparent, but the possible causes are worth discussing. It was seen that designs derived from pentagons and hexagons consistently outperformed designs derived from larger equiradial sets. The reason for this cannot be stated conclusively; however, it may be related to the distribution of design points over the region of interest. The design points of pentagon and hexagon designs are distributed rather evenly over the entire region of interest. On the other hand, as the size of the equiradial set increases the distribution of design points becomes more heavily weighted toward the periphery of the region of interest. This weighting effect could also explain the observation that equiradial combination designs perform best when configured such that $n_2 > n_1$. It was also observed that the best designs among those derived from equiradial pentagons and hexagons were the uniform precision designs. This is intuitively agreeable with the development of the uniform precision criteria. The moderate value of λ_4 for these designs does not admit heavy bias effects, hence, performance should be comparable to that of minimum bias designs. Moreover, the uniform precision criterion implies that variance of the predictor is constant across the region of investigation. Since the true optimum was within this region for all designs considered, the results should be anticipated.

CHAPTER V

CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusions

Several conclusions can be drawn from this investigation. The best designs of the equiradial class are those derived from equiradial pentagons and hexagons. These constitute a valuable group of designs which generally outperform the more widely used central composite designs. Among designs derived from equiradial pentagons and hexagons, the best are the uniform precision designs. In designs constructed by the combination of equiradial sets, the best configuration is for the set with the largest number of points to be the interior set.

5.2 Recommendations

The results of this study indicate that further study of the class of equiradial designs would be of value. In the case of equiradial designs in two variables, design generation was easily accomplished by applying a simple trigonometric relationship. For the case of more than two variables, design construction becomes more complicated. The only equiradial designs for k greater than two which are found in the literature are the icosahedron and the dodecahedron (see Box and Hunter (3)), both for three variables. Box and Hunter point out that in five or more dimensions there are only three regular figures. They show that second order rotatable designs can be obtained by combining two of these figures, the cross-polytope and the hypercube. The resulting design can be seen to

be a member of the central composite class, however. Further investigation is recommended into the construction and performance of equiradial experimental designs for three and four variables.

APPENDICES

APPENDIX A

REPRESENTATIVE DESIGN AND MOMENT MATRICES

Uniform Precision Pentagon

$$D = \begin{bmatrix} .921 & .000 \\ .285 & .876 \\ -.745 & .541 \\ -.745 & -.541 \\ .285 & -.876 \\ .000 & .000 \\ .000 & .000 \\ .000 & .000 \end{bmatrix}$$

$$N^{-1}(X'X) = \begin{bmatrix} 1.000 & .000 & .000 & .265 & .265 & .000 \\ .000 & .265 & .000 & .000 & .000 & .000 \\ .000 & .000 & .265 & .000 & .000 & .000 \\ .265 & .000 & .000 & .169 & .056 & .000 \\ .265 & .000 & .000 & .056 & .169 & .000 \\ .000 & .000 & .000 & .000 & .000 & .056 \end{bmatrix}$$

Uniform Precision Hexagon

$$D = \begin{bmatrix} .892 & .000 \\ .446 & .773 \\ -.446 & .772 \\ -.892 & .000 \\ -.446 & -.773 \\ .446 & -.772 \\ .000 & .000 \\ .000 & .000 \\ .000 & .000 \end{bmatrix}$$

$$N^{-1}(X'X) = \begin{bmatrix} 1.000 & .000 & .000 & .265 & .265 & .000 \\ .000 & .265 & .000 & .000 & .000 & .000 \\ .000 & .000 & .265 & .000 & .000 & .000 \\ .265 & .000 & .000 & .158 & .053 & .000 \\ .265 & .000 & .000 & .053 & .158 & .000 \\ .000 & .000 & .000 & .000 & .000 & .053 \end{bmatrix}$$

Orthogonal Pentagon

$$D = \begin{bmatrix} 1.030 & .000 \\ .318 & .980 \\ -.833 & .605 \\ -.833 & -.605 \\ .318 & -.980 \\ .000 & .000 \\ .000 & .000 \\ .000 & .000 \\ .000 & .000 \\ .000 & .000 \end{bmatrix}$$

$$N^{-1}(X'X) = \begin{bmatrix} 1.000 & .000 & .000 & .265 & .265 & .000 \\ .000 & .265 & .000 & .000 & .000 & .000 \\ .000 & .000 & .265 & .000 & .000 & .000 \\ .265 & .000 & .000 & .211 & .070 & .000 \\ .265 & .000 & .000 & .070 & .211 & .000 \\ .000 & .000 & .000 & .000 & .000 & .070 \end{bmatrix}$$

Orthogonal Combination: Radial Pentagon/Interior Hexagon

$$D = \begin{bmatrix} 1.054 & .000 \\ .326 & 1.003 \\ -.853 & .620 \\ -.853 & -.620 \\ .326 & -1.003 \\ .215 & .000 \\ .107 & .186 \\ -.107 & .186 \\ -.215 & .000 \\ -.107 & -.186 \\ .107 & -.186 \end{bmatrix}$$

$$N^{-1}(X'X) = \begin{bmatrix} 1.000 & .000 & .000 & .265 & .265 & .000 \\ .000 & .265 & .000 & .000 & .000 & .000 \\ .000 & .000 & .265 & .000 & .000 & .000 \\ .265 & .000 & .000 & .211 & .070 & .000 \\ .265 & .000 & .000 & .070 & .211 & .000 \\ .000 & .000 & .000 & .000 & .000 & .070 \end{bmatrix}$$

Minimum Bias Hexagon

$$D = \begin{bmatrix} .841 & .000 \\ .420 & .728 \\ -.421 & .728 \\ -.841 & .000 \\ -.420 & -.728 \\ .421 & -.728 \\ .000 & .000 \\ .000 & .000 \end{bmatrix}$$

$$N^{-1}(X'X) = \begin{bmatrix} 1.000 & .000 & .000 & .265 & .265 & .000 \\ .000 & .265 & .000 & .000 & .000 & .000 \\ .000 & .000 & .265 & .000 & .000 & .000 \\ .265 & .000 & .000 & .141 & .047 & .000 \\ .265 & .000 & .000 & .047 & .141 & .000 \\ .000 & .000 & .000 & .000 & .000 & .047 \end{bmatrix}$$

Minimum Bias Combination: Radial Pentagon/Interior Pentagon

$$D = \begin{bmatrix} .891 & .000 \\ .275 & .847 \\ -.720 & .523 \\ -.720 & -.523 \\ .275 & -.847 \\ .517 & .000 \\ .160 & .492 \\ -.419 & .304 \\ -.419 & -.304 \\ .160 & -.492 \end{bmatrix}$$

$$N^{-1}(X'X) = \begin{bmatrix} 1.000 & .000 & .000 & .265 & .265 & .000 \\ .000 & .265 & .000 & .000 & .000 & .000 \\ .000 & .000 & .265 & .000 & .000 & .000 \\ .265 & .000 & .000 & .131 & .044 & .000 \\ .265 & .000 & .000 & .044 & .131 & .000 \\ .000 & .000 & .000 & .000 & .000 & .044 \end{bmatrix}$$

APPENDIX B

COMPUTER PROGRAM LISTING


```

CALL SURF(C,SURF,SP,AR,1,1,1)
IF (NSURF.EQ.4) RESP(ICP,IL)=AR(1)
IF (NSURF.EQ.5) RESP(ICP,IL)=1./ABS(AR(1))
IF (NSURF.EQ.6) RESP(ICP,IL)=AR(1)/4.173749909
213 CONTINUE
DO 205 I=1,16,3
  TSPD=0.0
  TRESP=0.0
  DO 204 J=1,30
    TSPD=TSPD+SPD(J,I)
    TRESP=TRESP+RESP(J,I)
204 CONTINUE
  ASPD(INSURF,I)=TSPD/30.0
  ARESP(INSURF,I)=TRESP/30.0
205 CONTINUE
  TSPD=0.0
  TRESP=0.0
  DO 206 I=1,16,3
    TSPD=TSPD+ASPD(I,SURF,I)
    TRESP=TRESP+ARESP(I,SURF,I)
206 CONTINUE
  OASPD=TSPD/6.0
  OARESP=TRESP/6.0
  WRITE(6,5)SURF,(ASPD(INSURF,I),I=1,16,3),OASPD,(ARESP(NSURF
1,I),I=1,16,3),OARESP
207 CONTINUE
  TOAS=0.0
  TOAR=0.0
  DO 301 I=1,16,3
    TSPD=0.0
    TRESP=0.0
    DO 300 J=1,6
      TSPD=TSPD+SPD(J,I)
      TRESP=TRESP+RESP(J,I)
300 CONTINUE
    TSPD(1)=TSPD/6.0
    TRESP(1)=TRESP/6.0
    TOAS=TOAS+TSPD(1)
    TOAR=TOAR+TRESP(1)
301 CONTINUE
    TOAS=TOAS/6.0
    TOAR=TOAR/6.0
    WRITE(6,6) (TSPD(1),I=1,16,3),TOAS,(TRESP(1),I=1,16,3),TOAR
5 FORMAT(11F10.5,'AVG SP DEV',T33,7F10.5//,T25,'AVG RESP',
1T33,7F10.5)
6 FORMAT(11F10.5,'ALL',T25,'AVG SP DEV',T33,7F10.5//,T25,'AVG RESP',
1T33,7F10.5)
GO TO 400
1 WRITE(6,2)
400 CONTINUE
2 FORMAT(11F10.5,'SINGULAR MATRIX OR OVERFLOW ERROR')
STOP
END

```

```

SUBROUTINE SURF(NSURF,A,R,IN,MA,MB)
DIMENSION A(MA,2),R(MB)
GO TO (1,2,3,4,5,6),NSURF
1 DO 10 I=1,IN
  B(1)=((.5+.5*A(I,1))**.5)*A(I,2)**4.*EXP(2.-(.5+.5*A(I,1))**.5.-
1A(I,2)**4.)
10 CONTINUE
GO TO 7
2 DO 20 I=1,IN
  B(1)=((.7+.4*A(I,1)+.3*A(I,2))**.4)*((.8-.6*A(I,1)+.8*A(I,2))**.4.-
1*EXP(2.-(.5+.4*A(I,1)+.3*A(I,2))**.4)-(.8-.6*A(I,1)+.8*A(I,2))**.4.)
20 CONTINUE
GO TO 7
3 DO 30 I=1,IN
  B(1)=A(I,1)**2.*EXP(1.-A(I,1)**2.-20.25*(A(I,1)-A(I,2))**2.)
30 CONTINUE
GO TO 7
4 DO 40 I=1,IN
  B(1)=(.3*A(I,1)**2+.7*A(I,2)**2)**3.*EXP(1.-.6*(A(I,1)-A(I,2))**
12.-(.3*A(I,1)**2+.7*A(I,2)**2)**3.)
40 CONTINUE
GO TO 7
5 DO 50 I=1,IN
  B(1)=-(1.+100.*(A(I,2)-A(I,1)**2.))**2.+(1.-A(I,1))**2.)
50 CONTINUE
GO TO 7
6 DO 60 I=1,IN
  X=A(I,1)**2.
  Y=A(I,2)**2.
  Z=A(I,1)*A(I,2)
  B(1)=Z/(.9+.066*A(I,1)-.001*A(I,2)-.01*X+.03*Y+.0005*Z+.01*A(I,2)-
1-.017*A(I,1)*Y+.015*Z**2.)
60 CONTINUE
7 CONTINUE
RETURN
END

```

APPENDIX C

INDIVIDUAL DESIGN PERFORMANCE DATA

EQUILATRIAL DESIGN: 5 VERTICES AND 1 CENTER POINTS									
SURFACE	ST DEV	0.00	0.03	0.06	0.09	0.12	0.15	AVERAGE	
1	AVG SP D	.06375	.05602	.04377	.04344	.04070	.05229	.04999	
	AVG PESP	.97272	.98030	.98348	.99341	.99067	.98052	.98752	
2	AVG SP D	.05866	.05268	.05722	.06905	.08164	.09196	.08864	
	AVG PESP	.97574	.98266	.98119	.97690	.97155	.96578	.97564	
3	AVG SP D	.09087	.12246	.12815	.10023	.09995	.11709	.11129	
	AVG PESP	.94375	.92059	.91659	.94130	.94120	.92958	.93315	
4	AVG SP D	.61720	.58095	1.43651	0.18505	3.19941	1.15003	2.19761	
	AVG PESP	.78580	.83280	.87267	.87077	.89524	.88258	.85762	
5	AVG SP D	.33242	.40366	.42827	.45737	.49274	.53705	.45025	
	AVG PESP	.71281	.70345	.69318	.68138	.66933	.65004	.68595	
6	AVG SP D	.03123	.11583	.12886	.14199	.15864	.18009	.12021	
	AVG PESP	.99998	.99976	.99962	.99937	.99894	.99824	.99932	
ALL	AVG SP D	.20880	.22227	.37046	1.16029	.07980	.35025	.50067	
	AVG PESP	.89448	.90559	.90945	.91101	.91117	.90209	.90053	
EQUILATRIAL DESIGN: 5 VERTICES AND 2 CENTER POINTS									
SURFACE	ST DEV	0.00	0.03	0.06	0.09	0.12	0.15	AVERAGE	
1	AVG SP D	.06375	.04483	.04019	.04510	.05410	.06003	.05188	
	AVG PESP	.97272	.99162	.99434	.99078	.98489	.97029	.98544	
2	AVG SP D	.05866	.05043	.05509	.06758	.07030	.08011	.08035	
	AVG PESP	.97574	.98724	.98764	.98511	.98170	.97007	.98258	
3	AVG SP D	.09087	.09406	.09645	.10500	.09781	.11199	.10514	
	AVG PESP	.94375	.91020	.94234	.93045	.93953	.92002	.93451	
4	AVG SP D	.61719	.57772	.50058	.78747	.23230	.16229	.42959	
	AVG PESP	.78580	.83007	.86480	.81130	.80888	.94112	.85167	
5	AVG SP D	.33242	.41257	.49067	.49730	.56197	.65098	.49348	
	AVG PESP	.71281	.69937	.68422	.66692	.64020	.61077	.67105	
6	AVG SP D	.03123	.11710	.12965	.13785	.14389	.14007	.11805	
	AVG PESP	.99998	.99970	.99949	.99930	.99914	.99901	.99944	
ALL	AVG SP D	.20880	.25029	.21209	.27349	.19440	.20458	.22392	
	AVG PESP	.89848	.90536	.91214	.89831	.90339	.90701	.90412	
EQUILATRIAL DESIGN: 5 VERTICES AND 3 CENTER POINTS									
SURFACE	ST DEV	0.00	0.03	0.06	0.09	0.12	0.15	AVERAGE	
1	AVG SP D	.06375	.04460	.04135	.04834	.05871	.06002	.05422	
	AVG PESP	.97272	.99208	.99401	.98958	.98297	.97003	.98453	
2	AVG SP D	.05866	.05014	.05671	.06830	.07701	.08005	.08578	
	AVG PESP	.97574	.98794	.98820	.98531	.98147	.97037	.98267	
3	AVG SP D	.09087	.09089	.09541	.12579	.09262	.14918	.12663	
	AVG PESP	.94375	.91726	.94260	.92157	.93988	.90941	.92908	
4	AVG SP D	.61719	.24428	.36542	.29816	.15595	.12900	.30180	
	AVG PESP	.78580	.84704	.80738	.84906	.94599	.96924	.86753	
5	AVG SP D	.33242	.41456	.45467	.50719	.58057	.69098	.50557	
	AVG PESP	.71281	.69647	.68220	.66345	.64034	.60070	.68083	
6	AVG SP D	.03123	.11922	.13275	.14127	.14732	.15107	.12061	
	AVG PESP	.99998	.99966	.99942	.99921	.99905	.99892	.99937	
ALL	AVG SP D	.20880	.17028	.19105	.19817	.18530	.21208	.19577	
	AVG PESP	.89848	.90708	.90230	.90146	.91495	.90575	.90500	
EQUILATRIAL DESIGN: 5 VERTICES AND 4 CENTER POINTS									
SURFACE	ST DEV	0.00	0.03	0.06	0.09	0.12	0.15	AVERAGE	
1	AVG SP D	.06375	.04550	.04953	.04217	.04940	.05770	.04968	
	AVG PESP	.97272	.99107	.99452	.99186	.98072	.98073	.98027	
2	AVG SP D	.05866	.05078	.05564	.06716	.07003	.08003	.08520	
	AVG PESP	.97574	.98043	.98684	.98450	.98138	.97012	.98217	
3	AVG SP D	.09087	.09955	.09835	.09903	.14158	.09010	.13901	
	AVG PESP	.94375	.91007	.94130	.94031	.90952	.94048	.93201	
4	AVG SP D	.61719	1.05999	.28763	.41798	1.76740	.27004	.73816	
	AVG PESP	.78580	.93463	.89213	.80117	.81409	.84904	.82782	
5	AVG SP D	.33242	.41057	.44476	.48780	.54469	.62009	.48249	
	AVG PESP	.71281	.70027	.68622	.67034	.65173	.62053	.67478	
6	AVG SP D	.03123	.11506	.12634	.13399	.13989	.14404	.11519	
	AVG PESP	.99998	.99973	.99956	.99940	.99925	.99911	.99950	
ALL	AVG SP D	.20880	.23023	.17539	.20812	.45318	.21097	.20496	
	AVG PESP	.89848	.90470	.91511	.89301	.89045	.89000	.90043	
EQUILATRIAL DESIGN: 5 VERTICES AND 5 CENTER POINTS									
SURFACE	ST DEV	0.00	0.03	0.06	0.09	0.12	0.15	AVERAGE	
1	AVG SP D	.06375	.05232	.04914	.05119	.05740	.06000	.05081	
	AVG PESP	.97272	.98067	.98177	.99185	.98870	.98002	.98585	
2	AVG SP D	.05866	.05300	.06001	.07425	.09030	.10003	.07389	
	AVG PESP	.97574	.98054	.97691	.98024	.98087	.94003	.98789	
3	AVG SP D	.09087	.11565	.28179	.10990	.10201	.10012	.13589	
	AVG PESP	.94375	.93298	.91402	.93304	.93092	.93030	.93330	
4	AVG SP D	.61719	.72291	.26470	.22748	.22232	.25004	.31002	
	AVG PESP	.78580	.82743	.84601	.88147	.88710	.87021	.89199	
5	AVG SP D	.33242	.40079	.42168	.44580	.47422	.58009	.43890	
	AVG PESP	.71281	.70479	.68606	.68035	.67014	.66000	.69016	
6	AVG SP D	.03123	.12000	.17406	.28515	2.24590	3.25213	1.01942	
	AVG PESP	.99998	.99968	.99974	.99952	.99921	.98001	.99445	
ALL	AVG SP D	.20880	.17068	.20773	.19896	.53203	.71404	.34010	
	AVG PESP	.89848	.90532	.90587	.90928	.87151	.87018	.89394	

EQUIRADIAL DESIGN: 6 VERTICES AND 1 CENTER POINTS							
SURFACE	ERR	ST	DEV	0.00	0.03	0.06	AVERAGE
1	AVG	SP	D	.03024	.03163	.04770	.06047
	AVG	PESP		.98934	.99000	.99226	.97074
2	AVG	SP	D	.03094	.04051	.06132	.08003
	AVG	PESP		.98731	.99093	.98871	.97593
3	AVG	SP	D	.19548	.10086	.08741	.08034
	AVG	PESP		.93448	.95770	.95295	.93579
4	AVG	SP	D	.11540	.07146	.07891	.07902
	AVG	PESP		.93320	.98142	.98763	.97005
5	AVG	SP	D	1.72077	1.02735	1.34169	1.18110
	AVG	PESP		.63207	.63092	.62971	.62077
6	AVG	SP	D	.02879	.13505	.15371	.16773
	AVG	PESP		.99098	.99926	.99584	.99843
ALL	AVG	SP	D	.35644	.23094	.20346	.25002
	AVG	PESP		.91273	.92004	.92502	.91685
EQUIRADIAL DESIGN: 6 VERTICES AND 2 CENTER POINTS							
SURFACE	ERR	ST	DEV	0.00	0.03	0.06	AVERAGE
1	AVG	SP	D	.03024	.03115	.04637	.06170
	AVG	PESP		.98934	.99307	.99264	.98038
2	AVG	SP	D	.03094	.04035	.06099	.07121
	AVG	PESP		.98731	.99071	.98867	.98016
3	AVG	SP	D	.19548	.10003	.08803	.08100
	AVG	PESP		.93448	.95788	.95362	.94806
4	AVG	SP	D	.11540	.07257	.06832	.07054
	AVG	PESP		.93320	.98072	.98771	.98415
5	AVG	SP	D	1.72059	1.02543	1.53424	2.52042
	AVG	PESP		.63207	.63097	.62981	.62853
6	AVG	SP	D	.02879	.13505	.15339	.16249
	AVG	PESP		.99098	.99928	.99885	.99860
ALL	AVG	SP	D	.35641	.23076	.20523	.24703
	AVG	PESP		.91273	.92592	.92523	.92181
EQUIRADIAL DESIGN: 6 VERTICES AND 3 CENTER POINTS							
SURFACE	ERR	ST	DEV	0.00	0.03	0.06	AVERAGE
1	AVG	SP	D	.03024	.02977	.04191	.05506
	AVG	PESP		.98934	.99555	.99392	.98942
2	AVG	SP	D	.03094	.04788	.06025	.06970
	AVG	PESP		.98731	.98934	.98757	.98483
3	AVG	SP	D	.19548	.11488	.09162	.08310
	AVG	PESP		.93448	.95045	.95650	.95203
4	AVG	SP	D	.11540	.08097	.07024	.07763
	AVG	PESP		.93320	.97009	.98682	.98013
5	AVG	SP	D	1.72063	1.04153	2.33082	1.09095
	AVG	PESP		.63207	.63119	.63026	.62920
6	AVG	SP	D	.02879	.13434	.15008	.17147
	AVG	PESP		.99098	.99935	.99886	.99851
ALL	AVG	SP	D	.35641	.24143	.20332	.25932
	AVG	PESP		.91273	.92499	.92566	.92346
EQUIRADIAL DESIGN: 6 VERTICES AND 4 CENTER POINTS							
SURFACE	ERR	ST	DEV	0.00	0.03	0.06	AVERAGE
1	AVG	SP	D	.03024	.03291	.04252	.06034
	AVG	PESP		.98934	.99272	.99019	.98308
2	AVG	SP	D	.03094	.04045	.06322	.07710
	AVG	PESP		.98731	.98478	.97905	.97137
3	AVG	SP	D	.19548	.13024	.10412	.09005
	AVG	PESP		.93448	.95769	.96064	.96040
4	AVG	SP	D	.11540	.14082	.13018	.12419
	AVG	PESP		.93320	.93044	.93921	.95450
5	AVG	SP	D	1.72064	1.23073	1.02813	1.04727
	AVG	PESP		.63207	.63160	.63110	.63050
6	AVG	SP	D	.02879	.15939	.16501	.16119
	AVG	PESP		.99098	.99032	.98690	.98903
ALL	AVG	SP	D	.35642	.24007	.20970	.20312
	AVG	PESP		.91273	.98726	.98951	.98992
EQUIRADIAL DESIGN: 6 VERTICES AND 5 CENTER POINTS							
SURFACE	ERR	ST	DEV	0.00	0.03	0.06	AVERAGE
1	AVG	SP	D	.03024	.03576	.05621	.08538
	AVG	PESP		.98934	.99120	.98600	.97211
2	AVG	SP	D	.03094	.04903	.06550	.08209
	AVG	PESP		.98731	.98282	.97402	.96117
3	AVG	SP	D	.19548	.13072	.11016	.09524
	AVG	PESP		.93448	.95067	.96117	.96212
4	AVG	SP	D	.11540	.19005	.18005	.16171
	AVG	PESP		.93320	.92252	.94856	.95717
5	AVG	SP	D	1.72071	1.79700	1.03117	1.12570
	AVG	PESP		.63207	.63172	.63134	.63093
6	AVG	SP	D	.02879	.360039	.38756	.27797
	AVG	PESP		.99098	.99040	.98787	.99430
ALL	AVG	SP	D	.35643	.21918	.20370	.27315
	AVG	PESP		.91273	.95424	.95816	.96031

EQUIRADIAL DESIGN: 6 VERTICES AND 6 CENTER POINTS									
SURFACE	ERR	ST	LEV	0.00	0.03	0.06	0.09	0.12	AVERAGE
1	AVG	SP	D	.03624	.03083	.06532	.10722	.17137	.11578
	AVG	PESP		.09934	.08970	.07081	.05063	.03070	.09310
2	AVG	SP	D	.03694	.04968	.06811	.08924	.11315	.08341
	AVG	PESP		.08731	.08102	.06804	.04956	.02002	.04781
3	AVG	SP	D	.19548	.14265	.11616	.10022	.08975	.12116
	AVG	PESP		.93048	.05046	.06115	.06309	.06342	.09068
4	AVG	SP	D	.11540	.03789	.04671	.04352	.04518	.05067
	AVG	PESP		.03320	.01417	.04524	.07010	.02094	.06093
5	AVG	SP	D	1.72070	1.63282	1.06948	1.03505	1.02002	1.04621
	AVG	PESP		.63207	.63182	.63154	.63123	.63089	.63135
6	AVG	SP	D	.02079	.02226	.02459	.02563	.02159	.02125
	AVG	PESP		.09096	.03942	.09595	.09091	.09716	.09778
ALL	AVG	SP	D	.05643	.03235	.03173	.04171	.03284	.05725
	AVG	PESP		.01273	.08027	.08725	.09442	.07167	.07992
EQUIRADIAL DESIGN: 7 VERTICES AND 1 CENTER POINTS									
SURFACE	ERR	ST	LEV	0.00	0.03	0.06	0.09	0.12	AVERAGE
1	AVG	SP	D	.03686	.04277	.05737	.07159	.08432	.08015
	AVG	PESP		.09220	.09140	.08799	.08225	.07549	.09024
2	AVG	SP	D	.03652	.04098	.06336	.07487	.08558	.08700
	AVG	PESP		.09132	.08708	.08414	.07911	.07343	.08005
3	AVG	SP	D	.12660	.09133	.07499	.07437	.07572	.07700
	AVG	PESP		.93486	.06307	.05955	.05289	.04579	.04906
4	AVG	SP	D	.12043	.08501	.08661	.10143	.11647	.10053
	AVG	PESP		.03111	.06798	.07348	.07031	.06370	.06039
5	AVG	SP	D	1.04012	2.05110	1.08030	13.25455	1.07328	3.44957
	AVG	PESP		.63350	.63065	.62740	.62373	.61964	.62502
6	AVG	SP	D	.02226	.02068	.02429	.02578	.02693	.02131
	AVG	PESP		.09098	.09076	.09813	.09760	.09759	.09829
ALL	AVG	SP	D	.03230	.04581	.06752	2.30610	.08417	.07228
	AVG	PESP		.01268	.02331	.02178	.01708	.01261	.01702
EQUIRADIAL DESIGN: 7 VERTICES AND 2 CENTER POINTS									
SURFACE	ERR	ST	LEV	0.00	0.03	0.06	0.09	0.12	AVERAGE
1	AVG	SP	D	.03686	.04297	.05876	.07399	.08887	.10009
	AVG	PESP		.09228	.08099	.08435	.07076	.06699	.09328
2	AVG	SP	D	.03652	.04054	.06349	.07591	.08665	.08761
	AVG	PESP		.09132	.08403	.07886	.07133	.06258	.09311
3	AVG	SP	D	.12660	.12075	.07885	.07521	.07594	.09255
	AVG	PESP		.93486	.04373	.06306	.05708	.04962	.04071
4	AVG	SP	D	.12043	.10027	.11138	.12090	.15123	.13416
	AVG	PESP		.03111	.05442	.06026	.05060	.04483	.04539
5	AVG	SP	D	1.04012	1.05013	1.75107	1.08794	2.00759	1.39854
	AVG	PESP		.63350	.63074	.62771	.62439	.62079	.62008
6	AVG	SP	D	.02227	1.15286	9.91969	13.49800	7.31135	3.22076
	AVG	PESP		.09098	.08497	.08658	.07401	.06590	.04799
ALL	AVG	SP	D	.03230	.04559	1.09721	2.48976	1.63027	1.20922
	AVG	PESP		.01268	.01430	.02214	.03536	.04179	.02006
EQUIRADIAL DESIGN: 7 VERTICES AND 3 CENTER POINTS									
SURFACE	ERR	ST	LEV	0.00	0.03	0.06	0.09	0.12	AVERAGE
1	AVG	SP	D	.03686	.04966	.08184	.12842	.19765	.31037
	AVG	PESP		.09228	.08068	.05892	.04230	.02530	.09301
2	AVG	SP	D	.03652	.05052	.07102	.09525	.12460	.16041
	AVG	PESP		.09132	.07589	.05439	.02000	.07100	.06024
3	AVG	SP	D	.12660	.04001	.02500	.09943	.08980	.11713
	AVG	PESP		.93486	.07084	.06657	.05304	.04192	.04256
4	AVG	SP	D	.12043	.04159	.05101	.05002	.07010	53.15107
	AVG	PESP		.03111	.04118	.05128	.07040	.07308	.04448
5	AVG	SP	D	1.04012	1.04260	1.07527	1.16122	1.30571	1.27583
	AVG	PESP		.63350	.63089	.62817	.62535	.62244	.62003
6	AVG	SP	D	.02227	.02079	.01475	.01308	.01484	.01005
	AVG	PESP		.09098	.09090	.09003	.09887	.09872	.09905
ALL	AVG	SP	D	.03230	.06103	.05491	.07217	.03864	1.04518
	AVG	PESP		.01268	.07010	.07739	.06619	.02209	.05015
EQUIRADIAL DESIGN: 7 VERTICES AND 4 CENTER POINTS									
SURFACE	ERR	ST	LEV	0.00	0.03	0.06	0.09	0.12	AVERAGE
1	AVG	SP	D	.03686	.05249	.09166	.15454	.26645	.54977
	AVG	PESP		.09228	.07030	.04876	.03001	.03079	.04048
2	AVG	SP	D	.03652	.05152	.07418	.10305	.14143	.19003
	AVG	PESP		.09132	.07349	.04631	.02097	.03151	.07032
3	AVG	SP	D	.12660	.03178	.01559	.01538	.01500	.01033
	AVG	PESP		.93486	.04112	.04659	.04049	.03253	.01058
4	AVG	SP	D	.12043	.01499	.04458	.04241	2.02482	3.92726
	AVG	PESP		.03111	.04001	.04637	.04832	.05741	.04028
5	AVG	SP	D	1.04012	1.04086	1.04400	1.05013	1.08420	1.00091
	AVG	PESP		.63350	.63091	.62826	.62553	.62273	.62080
6	AVG	SP	D	.02227	.01933	.01656	.01911	.01719	.01000
	AVG	PESP		.09098	.09044	.09027	.09913	.09900	.09927
ALL	AVG	SP	D	.03230	.05014	.04008	.03044	.03220	.01128
	AVG	PESP		.01268	.07051	.06759	.04334	.02009	.03202

EQUATORIAL DESIGN: 7 VERTICES AND 5 CENTER POINTS								
SURFACE	ERR	ST DEV	0.00	0.03	0.06	0.09	0.12	0.15
1	AVG	SP D	.03480	.05097	.10125	.18331	.36120	13.23003
	AVG	PESP	.99828	.97018	.97650	.84121	.64530	.44012
2	AVG	SP D	.03452	.05246	.07730	.11006	.15901	.23041
	AVG	PESP	.99832	.97139	.93879	.88101	.79219	.67471
3	AVG	SP D	.12660	.15052	.28561	.22301	.16793	.12205
	AVG	PESP	.93480	.91188	.89836	.87587	.87013	.86002
4	AVG	SP D	.12043	.42135	.43302	2.08035	1.97401	1837.10236
	AVG	PESP	.93111	.78971	.80330	.71267	.50793	.46010
5	AVG	SP D	1.04012	1.04406	1.04245	1.04141	1.04170	1.04023
	AVG	PESP	.63350	.63093	.62832	.62507	.62297	.62042
6	AVG	SP D	.02027	.16713	.15551	.15935	.16163	.16400
	AVG	PESP	.99090	.99946	.99942	.99932	.99920	.99910
ALL	AVG	SP D	.23230	.31058	.34069	.63438	.64425	308.05037
	AVG	PESP	.91260	.87993	.86780	.82210	.73962	.68038
EQUATORIAL DESIGN: 7 VERTICES AND 6 CENTER POINTS								
SURFACE	ERR	ST DEV	0.00	0.03	0.06	0.09	0.12	0.15
1	AVG	SP D	.03480	.04064	.07843	.11994	.17810	.26775
	AVG	PESP	.99828	.98157	.96232	.92204	.85149	.74049
2	AVG	SP D	.03452	.05017	.06995	.09209	.11913	.15102
	AVG	PESP	.99832	.97080	.95730	.92736	.88439	.82092
3	AVG	SP D	.12660	.23031	.14267	.09325	.08664	.08715
	AVG	PESP	.93480	.88085	.92006	.95024	.94392	.92043
4	AVG	SP D	.12043	1.56017	.37227	.39132	.50048	2.13048
	AVG	PESP	.93111	.79308	.81031	.80007	.70734	.56406
5	AVG	SP D	1.04012	1.04061	1.10016	1.25402	1.75465	45.23420
	AVG	PESP	.63350	.63097	.62814	.62528	.62232	.61924
6	AVG	SP D	.02027	.21475	.19331	.19107	.19160	.19208
	AVG	PESP	.99098	.99694	.99890	.99874	.99860	.99848
ALL	AVG	SP D	.23230	.52078	.32613	.35713	.48177	8.01008
	AVG	PESP	.91260	.87788	.87952	.87176	.83467	.77074
EQUATORIAL DESIGN: 7 VERTICES AND 7 CENTER POINTS								
SURFACE	ERR	ST DEV	0.00	0.03	0.06	0.09	0.12	0.15
1	AVG	SP D	.03480	.04938	.08887	.12598	.19190	.30076
	AVG	PESP	.99828	.98093	.95991	.91536	.83298	.70719
2	AVG	SP D	.03452	.05042	.07071	.09449	.12308	.15904
	AVG	PESP	.99832	.97014	.95521	.92203	.87488	.81075
3	AVG	SP D	.12660	.32024	.17860	.09747	.08883	.08002
	AVG	PESP	.93480	.88063	.89868	.95456	.94252	.92007
4	AVG	SP D	.12043	.44269	.41933	.46000	.62451	2.41244
	AVG	PESP	.93111	.79187	.81483	.79986	.68311	.53005
5	AVG	SP D	1.04012	1.04330	1.08120	1.18247	1.43205	2.41070
	AVG	PESP	.63350	.63088	.62816	.62533	.62240	.61907
6	AVG	SP D	.02027	.20007	.18695	.18500	.18660	.18000
	AVG	PESP	.99098	.99906	.99900	.99883	.99869	.99857
ALL	AVG	SP D	.23230	.35295	.33620	.35007	.44116	.92708
	AVG	PESP	.91260	.87792	.87597	.86942	.82576	.76023
EQUATORIAL DESIGN: 8 VERTICES AND 1 CENTER POINTS								
SURFACE	ERR	ST DEV	0.00	0.03	0.06	0.09	0.12	0.15
1	AVG	SP D	.03443	.04516	.06100	.07951	.10005	.12294
	AVG	PESP	.99842	.98215	.97052	.95279	.92810	.89005
2	AVG	SP D	.03457	.04588	.06070	.07743	.09574	.11590
	AVG	PESP	.99840	.98141	.96947	.95197	.92823	.89703
3	AVG	SP D	.06001	.06139	.06673	.07333	.08072	.08904
	AVG	PESP	.96090	.96112	.95856	.95224	.94109	.92399
4	AVG	SP D	.11439	.12977	.17275	.37819	.39055	.34209
	AVG	PESP	.93420	.91796	.87402	.82070	.80009	.78177
5	AVG	SP D	1.04008	1.05026	1.05286	1.05009	1.06168	1.06774
	AVG	PESP	.63348	.62073	.62300	.61900	.61403	.60099
6	AVG	SP D	.02029	.37438	.31019	.29028	.29115	.28038
	AVG	PESP	.99098	.99704	.99835	.99329	.99818	.99007
ALL	AVG	SP D	.22013	.28448	.28739	.32691	.33798	.33716
	AVG	PESP	.91750	.81155	.89914	.88250	.86924	.85008
EQUATORIAL DESIGN: 8 VERTICES AND 2 CENTER POINTS								
SURFACE	ERR	ST DEV	0.00	0.03	0.06	0.09	0.12	0.15
1	AVG	SP D	.03443	.06271	.11655	.22440	.32177	3.43023
	AVG	PESP	.99842	.96444	.90060	.75245	.52900	.34008
2	AVG	SP D	.03457	.06037	.10790	.19173	.38177	2.30475
	AVG	PESP	.99840	.96308	.90456	.77544	.58285	.42077
3	AVG	SP D	.06001	.05090	.08109	.08992	.27095	.41904
	AVG	PESP	.96090	.96088	.95012	.95098	.93758	.92004
4	AVG	SP D	.11439	.06098	.46147	1.19027	1.53202	.80097
	AVG	PESP	.93420	.77029	.76783	.62908	.44537	.48942
5	AVG	SP D	1.04008	1.43094	1.90514	1.25002	4.82001	1.23007
	AVG	PESP	.63348	.62088	.62436	.61994	.61563	.61141
6	AVG	SP D	.02028	.16300	.26149	1.00347	2.22088	.47003
	AVG	PESP	.99098	.99921	.99350	.90325	.49801	.95700
ALL	AVG	SP D	.22013	.37447	.47941	.84407	1.02040	1.44013
	AVG	PESP	.91750	.89308	.95836	.73911	.50475	.60005

EQUATORIAL DESIGN: 8 VERTICES AND 3 CENTER POINTS								
SURFACE\ERR	ST DEV	0.00	0.03	0.06	0.09	0.12	0.15	AVERAGE
1	AVG SP D	.03443	.06213	.11638	.21740	.40085	4.08020	.03425
	AVG RESP	.93442	.96493	.99321	.76109	.54221	.35979	.75327
2	AVG SP D	.03457	.05985	.10591	.18015	.30217	4.96922	.95298
	AVG RESP	.93440	.96446	.99602	.78553	.59495	.37704	.77963
3	AVG SP D	.06601	.05092	.08050	.97057	.32539	.34511	.30742
	AVG RESP	.96490	.96560	.95970	.85477	.83324	.81196	.81770
4	AVG SP D	.11439	.08038	.41929	1.93443	1.49932	.90770	.90925
	AVG RESP	.93420	.77003	.76725	.04513	.44888	.46715	.67311
5	AVG SP D	1.04408	1.41368	1.96743	1.25408	3.04907	1.28701	1.67053
	AVG RESP	.63440	.72037	.62435	.01792	.01559	.01136	.62226
6	AVG SP D	.02426	.16187	.24403	3.90140	4.00907	.53706	1.62362
	AVG RESP	.93440	.99425	.99497	.75527	.29717	.93006	.83078
ALL	AVG SP D	.22013	.78947	.48892	1.41108	1.70531	2.02253	1.04967
	AVG RESP	.91750	.89519	.85938	.73005	.55534	.60433	.75946
EQUATORIAL DESIGN: 8 VERTICES AND 4 CENTER POINTS								
SURFACE\ERR	ST DEV	0.00	0.03	0.06	0.09	0.12	0.15	AVERAGE
1	AVG SP D	.03443	.06294	.11942	.22723	.53094	2.75473	.02262
	AVG RESP	.93442	.96424	.99967	.74396	.52380	.33700	.74369
2	AVG SP D	.03457	.06058	.10871	.19400	.39003	2.17004	.49412
	AVG RESP	.93440	.96379	.99365	.77557	.57811	.42155	.77181
3	AVG SP D	.06601	.05090	.08247	1.06229	.20372	.73909	.51121
	AVG RESP	.96490	.96571	.95886	.84897	.84029	.82005	.90027
4	AVG SP D	.11439	.04246	.48773	1.19705	1.63087	.77432	.77467
	AVG RESP	.93420	.77039	.76768	.06677	.44195	.49979	.67113
5	AVG SP D	1.04408	1.44502	1.80390	1.26133	0.65240	1.21003	2.23840
	AVG RESP	.63348	.62088	.62436	.01995	.01564	.01443	.62229
6	AVG SP D	.02428	.16348	.26933	2.43275	2.12930	.45432	.91302
	AVG RESP	.99993	.99919	.99291	.85931	.58546	.96222	.89985
ALL	AVG SP D	.22013	.37223	.47859	1.19507	1.93390	1.35533	.92568
	AVG RESP	.91756	.88033	.85786	.74326	.59750	.60977	.76817
EQUATORIAL DESIGN: 8 VERTICES AND 5 CENTER POINTS								
SURFACE\ERR	ST DEV	0.00	0.03	0.06	0.09	0.12	0.15	AVERAGE
1	AVG SP D	.03443	.05116	.07995	.11841	.17543	.26052	.12083
	AVG RESP	.93442	.97473	.94643	.89468	.81101	.69077	.88567
2	AVG SP D	.03457	.05066	.07380	.10372	.15107	.22001	.10607
	AVG RESP	.93440	.97405	.94640	.89808	.82477	.72052	.89314
3	AVG SP D	.06601	.05983	.06816	.08379	.11053	.15058	.09015
	AVG RESP	.96490	.96553	.96181	.95042	.91786	.84700	.93372
4	AVG SP D	.11439	.08024	.59694	.58223	.74536	9.73006	2.07586
	AVG RESP	.93420	.78759	.78441	.75055	.68134	.55256	.74044
5	AVG SP D	1.04408	1.13573	1.40765	3.31506	2.12002	1.39594	1.73825
	AVG RESP	.63348	.62090	.62414	.01948	.01484	.01021	.62183
6	AVG SP D	.02428	.17309	.16223	.16129	.16281	.16541	.14219
	AVG RESP	.99993	.99945	.99940	.99940	.99940	.99931	.99952
ALL	AVG SP D	.22013	.75045	.39782	.72775	.57854	1.99005	.71222
	AVG RESP	.91756	.88003	.87711	.85221	.80821	.73920	.84705
EQUATORIAL DESIGN: 8 VERTICES AND 6 CENTER POINTS								
SURFACE\ERR	ST DEV	0.00	0.03	0.06	0.09	0.12	0.15	AVERAGE
1	AVG SP D	.03443	.05136	.07967	.11981	.17840	.27527	.12282
	AVG RESP	.93442	.97454	.94572	.89274	.80085	.69174	.88334
2	AVG SP D	.03457	.05081	.07428	.10680	.15330	.22533	.10752
	AVG RESP	.93440	.97396	.94573	.89096	.82124	.72000	.89113
3	AVG SP D	.06601	.05980	.06825	.08433	.11234	.16445	.09153
	AVG RESP	.96490	.96558	.96185	.95010	.91014	.84249	.93252
4	AVG SP D	.11439	.07082	.48794	.53322	.95187	3.01005	.93401
	AVG RESP	.93420	.78079	.78290	.75316	.68072	.54000	.74730
5	AVG SP D	1.04408	1.13909	1.43163	4.00202	1.95975	1.36914	1.82525
	AVG RESP	.63348	.62091	.62414	.01949	.01485	.01024	.62184
6	AVG SP D	.02428	.17105	.16132	.16050	.16217	.16496	.14153
	AVG RESP	.99993	.99946	.99950	.99940	.99940	.99931	.99952
ALL	AVG SP D	.22013	.73341	.33885	.82958	.58630	.86940	.53711
	AVG RESP	.91756	.88772	.87666	.85200	.80653	.73516	.84594
EQUATORIAL DESIGN: 8 VERTICES AND 7 CENTER POINTS								
SURFACE\ERR	ST DEV	0.00	0.03	0.06	0.09	0.12	0.15	AVERAGE
1	AVG SP D	.03443	.05012	.07581	.11118	.16031	.23503	.11091
	AVG RESP	.93442	.97578	.95020	.90431	.83283	.73007	.89798
2	AVG SP D	.03457	.05094	.07129	.10007	.13965	.19713	.09870
	AVG RESP	.93440	.97500	.95004	.90751	.84344	.75100	.90370
3	AVG SP D	.06601	.06000	.06770	.08110	.10227	.13498	.08435
	AVG RESP	.96490	.96325	.96153	.95150	.92557	.87212	.93910
4	AVG SP D	.11439	.08058	.50211	.51948	.52030	1.25527	.50735
	AVG RESP	.93420	.78001	.78097	.74738	.70110	.60094	.70209
5	AVG SP D	1.04408	1.11026	1.20975	1.92249	8.82607	1.72993	2.05700
	AVG RESP	.63348	.62080	.62411	.01942	.01474	.01006	.62177
6	AVG SP D	.02428	.18058	.16831	.16097	.16804	.16997	.14702
	AVG RESP	.99993	.99940	.99945	.99941	.99934	.99926	.99947
ALL	AVG SP D	.22013	.72473	.36416	.40350	1.05411	.61903	.61100
	AVG RESP	.91756	.89154	.87880	.85519	.81951	.76004	.85414

EQUATORIAL DESIGN: 8 VERTICES AND 1 CENTER POINTS									
SURFACE	ERR	ST	DEV	0.00	0.03	0.06	0.09	0.12	AVERAGE
1	AVG	SP	D	.03443	.04959	.07420	.10755	.15289	.21034
	AVG	PESP		.93442	.97035	.99233	.91017	.84390	.75439
2	AVG	SP	D	.03457	.04943	.07002	.09728	.13410	.18066
	AVG	PESP		.93440	.97065	.95196	.91254	.85299	.77318
3	AVG	SP	D	.06601	.06010	.06750	.07997	.09874	.12017
	AVG	PESP		.96020	.96309	.96135	.95203	.92869	.88215
4	AVG	SP	D	.11439	.06413	.36101	.38737	.50999	1.05272
	AVG	PESP		.93420	.91795	.77745	.75543	.71563	.62008
5	AVG	SP	D	1.04959	1.10452	1.25434	1.65049	5.09579	2.31431
	AVG	PESP		.63548	.62679	.62400	.61939	.61469	.60998
6	AVG	SP	D	.02620	.18065	.17251	.17101	.17198	.17316
	AVG	PESP		.99990	.99937	.99941	.99936	.99929	.99920
ALL	AVG	SP	D	.22613	.40307	.37326	.41561	1.02726	.77039
	AVG	PESP		.91756	.89353	.97777	.85816	.82586	.77336
EQUATORIAL DESIGN: 9 VERTICES AND 1 CENTER POINTS									
SURFACE	ERR	ST	DEV	0.00	0.03	0.06	0.09	0.12	AVERAGE
1	AVG	SP	D	.03462	.11015	.40337	1.031463	6.24988	1.33903
	AVG	PESP		.98839	.92915	.58452	.04310	.21531	.27930
2	AVG	SP	D	.03462	.12952	1.05690	.08998	.57423	2.05007
	AVG	PESP		.98839	.92746	.25572	.35474	.43052	.39408
3	AVG	SP	D	.06112	.23099	.11172	.13436	.08083	.17045
	AVG	PESP		.96585	.93432	.95979	.94506	.85464	.90315
4	AVG	SP	D	.11210	.73161	.86632	1.03206	35.25332	3.20011
	AVG	PESP		.93562	.72348	.56979	.41560	.44754	.39328
5	AVG	SP	D	1.04900	1.27304	1.33026	1.41350	26.77359	1.12379
	AVG	PESP		.63348	.62628	.62297	.61745	.61160	.60520
6	AVG	SP	D	.02829	3.73965	.26229	.22500	.21296	.20040
	AVG	PESP		.99998	.98184	.99348	.99587	.99650	.99009
ALL	AVG	SP	D	.21098	.96949	1.00514	3.30275	11.01080	1.45137
	AVG	PESP		.91862	.78742	.66438	.56207	.59403	.59302
EQUATORIAL DESIGN: 9 VERTICES AND 2 CENTER POINTS									
SURFACE	ERR	ST	DEV	0.00	0.03	0.06	0.09	0.12	AVERAGE
1	AVG	SP	D	.03462	.07045	.18464	.51398	6.17561	2.40006
	AVG	PESP		.98839	.95309	.93780	.48086	.04458	.08097
2	AVG	SP	D	.03462	.09021	.29794	3.33092	.67478	.48129
	AVG	PESP		.98839	.95513	.75713	.09492	.40093	.47393
3	AVG	SP	D	.06112	.16500	.25158	.20025	.19365	.25091
	AVG	PESP		.96585	.92320	.90487	.89034	.90083	.86371
4	AVG	SP	D	.11210	.75020	.36856	1.04006	0.38217	1.83043
	AVG	PESP		.93562	.73717	.66863	.51166	.38937	.47325
5	AVG	SP	D	1.04904	1.08903	1.04817	1.07836	1.80017	2.16001
	AVG	PESP		.63348	.62625	.62292	.61744	.61175	.60500
6	AVG	SP	D	.02829	4.12256	.36749	.26020	.23374	.22108
	AVG	PESP		.99998	.97044	.98222	.99379	.99554	.99022
ALL	AVG	SP	D	.21097	1.13358	.55306	1.07003	2.57669	1.21303
	AVG	PESP		.91862	.81178	.79560	.59917	.55717	.58205
EQUATORIAL DESIGN: 9 VERTICES AND 3 CENTER POINTS									
SURFACE	ERR	ST	DEV	0.00	0.03	0.06	0.09	0.12	AVERAGE
1	AVG	SP	D	.03462	.07230	.15776	.36119	9.01510	3.08205
	AVG	PESP		.98839	.95742	.86653	.61051	.24007	.04342
2	AVG	SP	D	.03462	.08259	.24065	4.10005	1.05714	.50036
	AVG	PESP		.98839	.95937	.81815	.18239	.35226	.48012
3	AVG	SP	D	.06112	.32779	.15755	.15088	.36009	.25009
	AVG	PESP		.96585	.90079	.92686	.93239	.89988	.87309
4	AVG	SP	D	.11210	.83070	.60459	.50509	1.34031	5.05326
	AVG	PESP		.93562	.73942	.68964	.58168	.40045	.40127
5	AVG	SP	D	1.04900	2.01434	1.43154	1.15336	1.23320	1.50003
	AVG	PESP		.63348	.62624	.62290	.61743	.61177	.60508
6	AVG	SP	D	.02828	.70522	.46205	.27309	.24377	.22308
	AVG	PESP		.99998	.94174	.96250	.99240	.99497	.99309
ALL	AVG	SP	D	.21097	.73891	.52418	1.09306	2.39160	1.77100
	AVG	PESP		.91862	.83591	.81444	.65283	.50324	.56108
EQUATORIAL DESIGN: 9 VERTICES AND 4 CENTER POINTS									
SURFACE	ERR	ST	DEV	0.00	0.03	0.06	0.09	0.12	AVERAGE
1	AVG	SP	D	.03462	.04992	.07323	.10003	.15209	.22143
	AVG	PESP		.98839	.97402	.94823	.90341	.83210	.72937
2	AVG	SP	D	.03462	.05343	.09473	.17101	.35250	1.90730
	AVG	PESP		.98839	.97759	.94041	.87708	.70250	.41005
3	AVG	SP	D	.06112	.18318	.24358	.36533	3.39010	.30425
	AVG	PESP		.96585	.90047	.97353	.87049	.30244	.85007
4	AVG	SP	D	.11210	.17773	1.15058	.57003	.55975	.69308
	AVG	PESP		.93562	.85088	.70297	.67525	.63225	.59302
5	AVG	SP	D	1.04904	1.51332	1.60745	1.25000	24.93130	1.20006
	AVG	PESP		.63348	.62820	.62283	.61737	.61181	.60012
6	AVG	SP	D	.02829	.21150	.24658	.52407	3.08096	3.58718
	AVG	PESP		.99998	.99096	.99712	.94815	.89832	.80172
ALL	AVG	SP	D	.21097	.76483	.56036	.50017	5.44048	1.32133
	AVG	PESP		.91862	.88902	.80901	.61039	.75057	.64400

EQUIDISTANT DESIGN: 9 VERTICES AND 9 CENTER POINTS									
SURFACE	ERR	ST DEV	0.00	0.03	0.06	0.09	0.12	0.15	AVERAGE
1	AVG SP D		.03462	.04956	.07238	.10417	.14649	.21442	.10394
	AVG RESP		.98639	.97465	.96908	.96558	.96377	.96309	.96976
2	AVG SP D		.03462	.05307	.09340	.16751	.33675	.26555	.50978
	AVG RESP		.98639	.97782	.95046	.88165	.71616	.43245	.82449
3	AVG SP D		.06112	.19645	.26279	1.04533	.65487	.58167	.53360
	AVG RESP		.96585	.99782	.86789	.67193	.55080	.45767	.88037
4	AVG SP D		.11210	.17366	.74213	.51934	.89390	8.03458	1.74593
	AVG RESP		.93562	.86158	.79200	.68064	.63285	.59076	.73528
5	AVG SP D		1.04604	1.49449	1.65271	1.24945	0.57412	1.23558	2.20920
	AVG RESP		.63348	.62020	.62283	.61737	.61181	.60612	.61997
6	AVG SP D		.02626	.21297	.24040	.43090	3.36706	2.92014	1.20096
	AVG RESP		.99698	.99097	.99750	.97580	.84647	.72501	.92362
ALL	AVG SP D		.21097	.76537	.51663	.68612	1.99586	2.60746	1.00390
	AVG RESP		.91862	.88984	.84829	.82220	.75015	.65958	.81475
EQUIDISTANT DESIGN: 9 VERTICES AND 9 CENTER POINTS									
SURFACE	ERR	ST DEV	0.00	0.03	0.06	0.09	0.12	0.15	AVERAGE
1	AVG SP D		.03462	.04777	.06653	.09154	.12473	.17058	.08926
	AVG RESP		.98639	.97641	.95519	.92571	.86842	.79553	.91739
2	AVG SP D		.03462	.05043	.08464	.14051	.24091	.52903	.16106
	AVG RESP		.98639	.97957	.95790	.90776	.79902	.59518	.67045
3	AVG SP D		.06112	.25671	.31616	.41793	.26354	.36001	.27963
	AVG RESP		.96585	.99540	.83712	.64300	.55235	.43007	.87330
4	AVG SP D		.11210	.14988	.23173	.95751	.85069	.56500	.46079
	AVG RESP		.93562	.88626	.78467	.67417	.65369	.60003	.75721
5	AVG SP D		1.04604	1.77497	2.35769	1.24072	1.04527	1.77957	1.57451
	AVG RESP		.63348	.62019	.62282	.61736	.61180	.60614	.61997
6	AVG SP D		.02626	.23148	.22062	.24337	.29977	.45077	.24672
	AVG RESP		.99698	.99092	.99870	.99708	.99494	.98202	.99547
ALL	AVG SP D		.21097	.35221	.54613	.51523	.57182	.64001	.47533
	AVG RESP		.91862	.89613	.85939	.82678	.79671	.73018	.83897
EQUIDISTANT DESIGN: 9 VERTICES AND 7 CENTER POINTS									
SURFACE	ERR	ST DEV	0.00	0.03	0.06	0.09	0.12	0.15	AVERAGE
1	AVG SP D		.03462	.04699	.06412	.08646	.11548	.15429	.08366
	AVG RESP		.98639	.97723	.95789	.92714	.86138	.78103	.92503
2	AVG SP D		.03462	.04924	.08091	.12902	.21696	.40709	.13309
	AVG RESP		.98639	.98037	.96095	.91800	.82864	.66719	.89092
3	AVG SP D		.06112	1.27263	.56116	1.77912	.26973	.74416	.76132
	AVG RESP		.96585	.96223	.85965	.83039	.85533	.83196	.87357
4	AVG SP D		.11210	.14152	.18921	.30595	.56007	.82541	.35539
	AVG RESP		.93562	.89706	.83194	.72032	.64581	.62455	.77585
5	AVG SP D		1.04604	1.72561	3.46415	1.26701	1.41095	3.41106	1.96987
	AVG RESP		.63348	.62019	.62282	.61736	.61180	.60614	.61996
6	AVG SP D		.02626	.24031	.22274	.22923	.25072	.28997	.21121
	AVG RESP		.99698	.99081	.99876	.99817	.99690	.99406	.99789
ALL	AVG SP D		.21097	.51438	.76356	.63303	.47165	.97193	.55956
	AVG RESP		.91862	.89655	.87200	.83523	.80332	.75741	.84720
EQUIDISTANT DESIGN: 9 VERTICES AND 8 CENTER POINTS									
SURFACE	ERR	ST DEV	0.00	0.03	0.06	0.09	0.12	0.15	AVERAGE
1	AVG SP D		.03462	.04625	.06805	.09479	.13076	.18121	.09295
	AVG RESP		.98639	.97593	.95353	.91671	.86021	.78055	.91255
2	AVG SP D		.03462	.05115	.08652	.14738	.26770	.64299	.20506
	AVG RESP		.98639	.97909	.95585	.90114	.77904	.63706	.89086
3	AVG SP D		.06112	.75117	1.99046	.50616	.29873	.31029	.58066
	AVG RESP		.96585	.99165	.85227	.84728	.84203	.83701	.87278
4	AVG SP D		.11210	.15552	.28420	.56557	.47900	.93002	.42110
	AVG RESP		.93562	.88212	.74865	.67576	.64540	.59407	.74707
5	AVG SP D		1.04604	1.40474	2.05765	1.23495	1.90509	1.51094	1.52840
	AVG RESP		.63348	.62019	.62282	.61736	.61180	.60613	.61997
6	AVG SP D		.02626	.22482	.22234	.26274	.38696	1.17223	.33524
	AVG RESP		.99698	.99096	.99856	.99079	.98762	.98113	.97717
ALL	AVG SP D		.21097	.37261	.79497	.46895	.57838	.79202	.53623
	AVG RESP		.91862	.89266	.85529	.82534	.78769	.70053	.83107
EQUIDISTANT DESIGN: 9 VERTICES AND 9 CENTER POINTS									
SURFACE	ERR	ST DEV	0.00	0.03	0.06	0.09	0.12	0.15	AVERAGE
1	AVG SP D		.03462	.04692	.06392	.08604	.11473	.15501	.09521
	AVG RESP		.98639	.97731	.95812	.92708	.86240	.78055	.92566
2	AVG SP D		.03462	.04914	.07967	.12893	.21461	.39970	.15111
	AVG RESP		.98639	.98044	.96122	.91304	.83100	.67501	.89243
3	AVG SP D		.06112	120.00745	.42730	.68135	.27359	.87004	20.53009
	AVG RESP		.96585	.99987	.86129	.63131	.65419	.82955	.87369
4	AVG SP D		.11210	.14055	.18664	.28940	.99200	.79593	.41950
	AVG RESP		.93562	.89775	.83098	.72714	.64499	.62579	.77738
5	AVG SP D		1.04604	1.72597	3.68473	1.27118	1.40385	3.00009	2.09529
	AVG RESP		.63348	.62019	.62282	.61736	.61180	.60614	.61996
6	AVG SP D		.02626	.24796	.22319	.22871	.24871	.28524	.21033
	AVG RESP		.99698	.99090	.99875	.99819	.99702	.99406	.99793
ALL	AVG SP D		.21097	20.45532	.77091	.44755	.54126	1.00204	3.91592
	AVG RESP		.91862	.89793	.87296	.83075	.80350	.75043	.84785

EQUATORIAL DESIGN: 10 VERTICES AND 1 CENTER POINTS								
SURFACE	ST	DEV	0.00	0.03	0.06	0.09	0.12	0.15
1	AVG	SP D	.03460	.06202	.13381	.31342	3.03758	2.24096
	AVG	PESP	.92839	.96982	.90265	.06754	.43999	.22007
2	AVG	SP D	.03462	.06024	.20218	2.96339	.99061	.39045
	AVG	PESP	.93040	.97078	.89378	.20729	.47950	.08005
3	AVG	SP D	.06043	.27091	.23969	3.23494	.45162	.24009
	AVG	PESP	.96602	.89734	.90330	.88093	.35088	.68007
4	AVG	SP D	.11240	.15023	.39730	.71200	.97053	2.00305
	AVG	PESP	.93540	.86162	.72047	.60412	.57085	.35026
5	AVG	SP D	1.04001	6.75742	1.10376	3.36290	1.08582	1.47421
	AVG	PESP	.63347	.62940	.67494	.62006	.61472	.60006
6	AVG	SP D	.02829	.33034	.25235	.23248	.22387	.21006
	AVG	PESP	.99098	.99172	.90662	.99077	.99082	.99003
ALL	AVG	SP D	.21490	1.20983	.40152	1.80417	1.12934	1.10097
	AVG	PESP	.91061	.89011	.84063	.66702	.66140	.65004
EQUATORIAL DESIGN: 10 VERTICES AND 2 CENTER POINTS								
SURFACE	ST	DEV	0.00	0.03	0.06	0.09	0.12	0.15
1	AVG	SP D	.03460	.06497	.14429	.37993	4.04345	2.32472
	AVG	PESP	.92839	.96094	.89053	.03392	.42761	.22043
2	AVG	SP D	.03462	.07229	.22885	21.11313	.71222	.36020
	AVG	PESP	.93040	.97102	.87016	.10306	.52712	.71045
3	AVG	SP D	.06043	.29903	.16409	.36202	.38149	.31006
	AVG	PESP	.96602	.80380	.92369	.91003	.80464	.68005
4	AVG	SP D	.11240	.17017	1.21237	.65902	2.09496	1.08096
	AVG	PESP	.93540	.85005	.69229	.60203	.58084	.55002
5	AVG	SP D	1.04001	89.90016	1.20872	1.83343	1.07096	2.60700
	AVG	PESP	.63347	.62940	.62492	.61999	.61456	.60009
6	AVG	SP D	.02829	.32025	.31542	.37399	.51578	.64002
	AVG	PESP	.99098	.99006	.99777	.99485	.98199	.98276
ALL	AVG	SP D	.21091	15.13965	.54562	4.11109	1.57115	1.25723
	AVG	PESP	.91061	.88774	.83323	.64500	.66713	.64007
EQUATORIAL DESIGN: 10 VERTICES AND 3 CENTER POINTS								
SURFACE	ST	DEV	0.00	0.03	0.06	0.09	0.12	0.15
1	AVG	SP D	.03460	.04017	.07418	.10899	.15449	.21701
	AVG	PESP	.92839	.98271	.96695	.93002	.88574	.80023
2	AVG	SP D	.03462	.04231	.07432	.13129	.23072	.49045
	AVG	PESP	.92840	.99034	.98122	.94352	.85597	.62021
3	AVG	SP D	.06043	.10030	.78697	.25236	2.51430	.31007
	AVG	PESP	.96602	.94991	.83804	.66103	.63665	.63003
4	AVG	SP D	.11240	.10255	.12106	.15007	.18014	.23702
	AVG	PESP	.93540	.92784	.99730	.65053	.60483	.74790
5	AVG	SP D	1.04001	1.23451	3.65452	1.30905	1.20287	1.32001
	AVG	PESP	.63347	.62944	.62521	.62077	.61013	.61027
6	AVG	SP D	.02829	.60046	.51884	.32019	.27340	.25001
	AVG	PESP	.99098	.95562	.97538	.99028	.99297	.99006
ALL	AVG	SP D	.21090	.35064	.87498	.41219	.70165	.47774
	AVG	PESP	.91061	.90598	.88070	.86891	.83205	.76078
EQUATORIAL DESIGN: 10 VERTICES AND 4 CENTER POINTS								
SURFACE	ST	DEV	0.00	0.03	0.06	0.09	0.12	0.15
1	AVG	SP D	.03460	.04946	.07855	.11909	.17790	.26900
	AVG	PESP	.92839	.98149	.96247	.92400	.85502	.74047
2	AVG	SP D	.03462	.04446	.08150	.15378	.31548	1.26007
	AVG	PESP	.92840	.98935	.97712	.93064	.77944	.32003
3	AVG	SP D	.06043	.12727	.34015	2.30900	.24041	.51002
	AVG	PESP	.96602	.93028	.85221	.83003	.87938	.82903
4	AVG	SP D	.11240	.10588	.12629	.16137	.20480	.27021
	AVG	PESP	.93540	.92401	.88821	.83751	.77443	.70005
5	AVG	SP D	1.04001	1.29051	7.31561	1.28823	1.22023	2.07777
	AVG	PESP	.63347	.62944	.62518	.62009	.61590	.61099
6	AVG	SP D	.02829	1.53465	.40028	.28234	.25046	.23028
	AVG	PESP	.99098	.90928	.96600	.99283	.99442	.99011
ALL	AVG	SP D	.21090	.62020	1.39106	.71905	.40357	.77006
	AVG	PESP	.91061	.96147	.98188	.85703	.81044	.70113
EQUATORIAL DESIGN: 10 VERTICES AND 5 CENTER POINTS								
SURFACE	ST	DEV	0.00	0.03	0.06	0.09	0.12	0.15
1	AVG	SP D	.03460	.04071	.07600	.11337	.16399	.23777
	AVG	PESP	.92839	.98219	.96508	.93107	.87334	.78477
2	AVG	SP D	.03462	.04303	.07730	.14030	.26088	.64005
	AVG	PESP	.92840	.98902	.97905	.94152	.82733	.52001
3	AVG	SP D	.06043	.11005	.40344	.30539	.29240	.48009
	AVG	PESP	.96602	.94065	.85060	.86001	.80425	.82477
4	AVG	SP D	.11240	.10394	.12322	.15505	.19017	.25021
	AVG	PESP	.93540	.92022	.89353	.84803	.79233	.72009
5	AVG	SP D	1.04000	1.25915	8.58410	1.38509	1.20074	1.49009
	AVG	PESP	.63347	.62944	.62510	.62074	.61000	.61015
6	AVG	SP D	.02828	.62001	.46621	.30133	.20018	.24000
	AVG	PESP	.99098	.91103	.98137	.99108	.99309	.99403
ALL	AVG	SP D	.21090	.79017	1.63688	.40017	.39000	.55002
	AVG	PESP	.91061	.90741	.88055	.80006	.82017	.74000

EQUATORIAL DESIGN: 10 VERTICES AND 6 CENTER POINTS							
SURFACE	ERR	ST DEV	0.00	0.03	0.06	0.09	AVERAGE
1	AVG SP D		.03400	.04058	.07155	.11229	.11088
	AVG RESP		.98339	.98232	.96154	.93287	.92263
2	AVG SP D		.03462	.04301	.07657	.13002	.19174
	AVG RESP		.98640	.98002	.97097	.94330	.88166
3	AVG SP D		.06043	.11110	.40105	.28511	.27049
	AVG RESP		.96602	.94037	.85311	.66145	.60442
4	AVG SP D		.11240	.10360	.12260	.15396	.15601
	AVG RESP		.93541	.92061	.89047	.84938	.80591
5	AVG SP D		1.04001	1.25000	6.32186	1.40982	2.11268
	AVG RESP		.63347	.62944	.62520	.62074	.61607
6	AVG SP D		.02828	.75067	.48108	.30546	.34721
	AVG RESP		.99098	.92095	.92010	.99130	.99446
ALL	AVG SP D		.21090	.78099	1.26147	.40078	.63283
	AVG RESP		.91861	.90022	.88303	.86659	.85804
EQUATORIAL DESIGN: 10 VERTICES AND 7 CENTER POINTS							
SURFACE	ERR	ST DEV	0.00	0.03	0.06	0.09	AVERAGE
1	AVG SP D		.03400	.05038	.09179	.12773	.13500
	AVG RESP		.98339	.98063	.95913	.91380	.89919
2	AVG SP D		.03462	.04000	.07697	.17309	7.13266
	AVG RESP		.98640	.98063	.97385	.91403	.80227
3	AVG SP D		.06043	.16299	.40014	.43752	.37556
	AVG RESP		.96602	.90164	.37209	.65028	.80434
4	AVG SP D		.11240	.10637	.13054	.16389	.17753
	AVG RESP		.93541	.92126	.89143	.82573	.83001
5	AVG SP D		1.04001	1.74073	2.86054	1.22041	25.53916
	AVG RESP		.63347	.62943	.62515	.62003	.61584
6	AVG SP D		.02828	22.76124	.35629	.26531	3.91308
	AVG RESP		.99098	.99477	.98947	.99390	.99508
ALL	AVG SP D		.21090	4.01245	.72088	.39999	6.21550
	AVG RESP		.91861	.83006	.84352	.85264	.87005
EQUATORIAL DESIGN: 10 VERTICES AND 8 CENTER POINTS							
SURFACE	ERR	ST DEV	0.00	0.03	0.06	0.09	AVERAGE
1	AVG SP D		.03460	.04937	.07825	.11885	.12048
	AVG RESP		.98339	.98157	.96278	.92490	.91128
2	AVG SP D		.03462	.04432	.09100	.15215	.20120
	AVG RESP		.98640	.98941	.97741	.93198	.83722
3	AVG SP D		.06043	.12524	.36925	.24605	.64224
	AVG RESP		.96602	.93091	.85296	.63096	.80285
4	AVG SP D		.11240	.10566	.12593	.16002	.16450
	AVG RESP		.93541	.92427	.89883	.83975	.84481
5	AVG SP D		1.04001	1.29112	0.02826	1.29009	2.04205
	AVG RESP		.63347	.62944	.62518	.62009	.61601
6	AVG SP D		.02828	1.39096	.41007	.28425	.43424
	AVG RESP		.99098	.94555	.98567	.99270	.99506
ALL	AVG SP D		.21090	.80161	1.60213	.74007	.71412
	AVG RESP		.91861	.86786	.80214	.85706	.84183
EQUATORIAL DESIGN: 10 VERTICES AND 9 CENTER POINTS							
SURFACE	ERR	ST DEV	0.00	0.03	0.06	0.09	AVERAGE
1	AVG SP D		.03460	.04773	.07271	.10541	.10157
	AVG RESP		.98339	.98317	.96853	.94112	.93404
2	AVG SP D		.03462	.04153	.07183	.12415	.14995
	AVG RESP		.98640	.99070	.98259	.95392	.91323
3	AVG SP D		.06043	.10087	.38894	.24302	.27063
	AVG RESP		.96602	.95236	.86004	.67039	.60073
4	AVG SP D		.11240	.10138	.11926	.14715	.14789
	AVG RESP		.93541	.92924	.90066	.86102	.80760
5	AVG SP D		1.04001	1.21465	2.60815	1.69001	1.50021
	AVG RESP		.63347	.62944	.62522	.62001	.61609
6	AVG SP D		.02828	.89154	.63915	.34151	.34146
	AVG RESP		.99098	.97291	.96637	.98376	.98559
ALL	AVG SP D		.21090	.73295	.65001	.44214	.41962
	AVG RESP		.91861	.90962	.89390	.87200	.86033
EQUATORIAL DESIGN: 10 VERTICES AND 10 CENTER POINTS							
SURFACE	ERR	ST DEV	0.00	0.03	0.06	0.09	AVERAGE
1	AVG SP D		.03460	.04705	.07027	.10940	.10392
	AVG RESP		.98339	.98401	.97132	.94847	.94387
2	AVG SP D		.03462	.04008	.06705	.11235	.12505
	AVG RESP		.98640	.99135	.98489	.96246	.93003
3	AVG SP D		.06043	.09386	.27810	.30308	.34075
	AVG RESP		.96602	.95520	.83802	.64078	.67424
4	AVG SP D		.11240	.09920	.11612	.14138	.14094
	AVG RESP		.93541	.93181	.90662	.87305	.87032
5	AVG SP D		1.04001	1.18123	1.80081	2.03339	1.54029
	AVG RESP		.63347	.62945	.62525	.62000	.61600
6	AVG SP D		.02828	.75984	1.23127	.40200	.43770
	AVG RESP		.99098	.98000	.98547	.98408	.97307
ALL	AVG SP D		.21090	.70356	.60134	.41024	.44045
	AVG RESP		.91861	.91514	.89850	.87202	.86043

UNIFORM PRECISION COMBINATION DESIGN: 5 RADIAL AND 5 INNER VERTICES R2/R1 = .374									
SURFACE	ERR	ST	DEV	0.00	0.03	0.06	0.09	0.12	0.15
AVERAGE									
1	AVG	SP	D	.07797	.06044	.05556	.05079	.00180	.00741
	AVG	PESP		.09114	.08110	.08887	.09032	.08743	.08431
2	AVG	SP	D	.07703	.06078	.06508	.07030	.09201	.10721
	AVG	PESP		.09030	.07330	.07160	.06527	.09034	.09447
3	AVG	SP	D	.11380	.11437	.24983	.16015	.11974	.12215
	AVG	PESP		.094794	.04780	.09731	.03030	.09098	.09403
4	AVG	SP	D	.09773	.1016	.25072	.23751	.22422	.22079
	AVG	PESP		.09987	.09799	.08844	.07296	.09044	.09070
5	AVG	SP	D	.40288	.02775	.45741	.49355	.53902	.59008
	AVG	PESP		.09942	.08779	.67551	.66201	.65008	.63105
6	AVG	SP	D	.03120	.12096	.16015	.21949	.38765	5.72037
	AVG	PESP		.09990	.09965	.09902	.09038	.09745	.07040
ALL	AVG	SP	D	.23162	.18474	.20796	.20338	.23741	1.14090
	AVG	PESP		.68712	.09794	.89512	.90429	.90079	.85006
UNIFORM PRECISION COMBINATION DESIGN: 5 RADIAL AND 5 INNER VERTICES R2/R1 = .413									
SURFACE	ERR	ST	DEV	0.00	0.03	0.06	0.09	0.12	0.15
AVERAGE									
1	AVG	SP	D	.07660	.06096	.06707	.07033	.09403	.12007
	AVG	PESP		.09505	.07084	.08342	.08033	.06980	.09402
2	AVG	SP	D	.07719	.06980	.08251	.10196	.12555	.15055
	AVG	PESP		.09122	.06070	.09418	.09044	.09139	.07942
3	AVG	SP	D	.14667	.12490	.11692	.12737	.17347	.07479
	AVG	PESP		.09040	.04044	.04534	.09046	.09102	.09148
4	AVG	SP	D	1.00200	.01700	.32061	.58149	.26200	.29773
	AVG	PESP		.73015	.79030	.82704	.85500	.87221	.88207
5	AVG	SP	D	.41780	.44327	.47369	.51092	.55000	.60246
	AVG	PESP		.09331	.08195	.07019	.05808	.04783	.03793
6	AVG	SP	D	.03119	.15016	.29394	1.16332	4.14760	.09226
	AVG	PESP		.09098	.09926	.09277	.091074	.09561	.06013
ALL	AVG	SP	D	.29792	.21266	.22570	.42030	.89340	.46019
	AVG	PESP		.68180	.09206	.89549	.88084	.83311	.05107
UNIFORM PRECISION COMBINATION DESIGN: 5 RADIAL AND 7 INNER VERTICES R2/R1 = .438									
SURFACE	ERR	ST	DEV	0.00	0.03	0.06	0.09	0.12	0.15
AVERAGE									
1	AVG	SP	D	.07035	.07369	.07712	.09043	.11560	.15009
	AVG	PESP		.09512	.07150	.07700	.07144	.09158	.09071
2	AVG	SP	D	.07627	.07766	.09579	.12298	.15542	.19021
	AVG	PESP		.09026	.09077	.09432	.091387	.088405	.08006
3	AVG	SP	D	.16620	.17913	.50570	.14067	.12851	.13474
	AVG	PESP		.09020	.09164	.091069	.090741	.091029	.09015
4	AVG	SP	D	.09002	.15015	.45826	.31963	.36750	.28095
	AVG	PESP		.72341	.76072	.80469	.83554	.85271	.86121
5	AVG	SP	D	.43134	.45220	.47638	.50488	.53913	.58173
	AVG	PESP		.09007	.07917	.07032	.06170	.05345	.04032
6	AVG	SP	D	.03118	.16046	.33304	1.58046	3.56217	.03003
	AVG	PESP		.09098	.09912	.09145	.07009	.53894	.09099
ALL	AVG	SP	D	.28724	.21088	.32438	.45984	.77800	.33213
	AVG	PESP		.67319	.08082	.88290	.86094	.79984	.84404
UNIFORM PRECISION COMBINATION DESIGN: 5 RADIAL AND 9 INNER VERTICES R2/R1 = .455									
SURFACE	ERR	ST	DEV	0.00	0.03	0.06	0.09	0.12	0.15
AVERAGE									
1	AVG	SP	D	.08161	.06540	.06040	.06727	.09048	.12101
	AVG	PESP		.09569	.07727	.08646	.08434	.09520	.09595
2	AVG	SP	D	.07883	.07332	.08963	.10703	.12088	.14006
	AVG	PESP		.09570	.06027	.095428	.094272	.09269	.09123
3	AVG	SP	D	.11019	.14389	2.90496	.12007	.14036	.27145
	AVG	PESP		.09384	.02061	.090438	.092002	.090473	.086904
4	AVG	SP	D	1.12030	.34707	.30651	.34713	.62192	.90109
	AVG	PESP		.71023	.76204	.81692	.81391	.77082	.73731
5	AVG	SP	D	.44577	.07921	.52082	.57408	.64859	.76041
	AVG	PESP		.09325	.07147	.66007	.04912	.03824	.02749
6	AVG	SP	D	.03110	.14426	.22530	.32723	.44770	.56124
	AVG	PESP		.09098	.09905	.09054	.09199	.08501	.09719
ALL	AVG	SP	D	.31281	.20686	.60960	.25000	.34000	.46009
	AVG	PESP		.87479	.08179	.88643	.86386	.80096	.84004
UNIFORM PRECISION COMBINATION DESIGN: 5 RADIAL AND 9 INNER VERTICES R2/R1 = .466									
SURFACE	ERR	ST	DEV	0.00	0.03	0.06	0.09	0.12	0.15
AVERAGE									
1	AVG	SP	D	.08257	.06769	.06058	.06252	.07439	.09442
	AVG	PESP		.09059	.07312	.08428	.08723	.08411	.07595
2	AVG	SP	D	.08101	.07262	.08145	.09007	.11328	.13001
	AVG	PESP		.09300	.06102	.09175	.09000	.09478	.09003
3	AVG	SP	D	.12013	.11030	.12070	.17335	.15130	.30009
	AVG	PESP		.09305	.03575	.093229	.09205	.09045	.08047
4	AVG	SP	D	1.70000	.44161	.05811	.03909	.04712	5.27020
	AVG	PESP		.70036	.74098	.74960	.07100	.00764	.07027
5	AVG	SP	D	.46140	.48093	.51605	.55315	.59803	.65092
	AVG	PESP		.07879	.07096	.66331	.05093	.04873	.04131
6	AVG	SP	D	.04114	.13147	.18835	.24047	.30447	.30129
	AVG	PESP		.09000	.09024	.09762	.09024	.09420	.09008
ALL	AVG	SP	D	.41300	.21944	.32102	.20334	.30470	1.13010
	AVG	PESP		.87073	.08081	.88140	.85909	.85784	.85132

UNIFORM PRECISION COMBINATION DESIGN:				6 RADIAL AND 6 INNER VERTICES R2/R1= .475				AVERAGE
SURFACE	ERR	ST DEV	0.00	0.03	0.06	0.09	0.12	0.15
1	AVG	SP D	.00029	.07000	.06672	.07430	.06939	.10705
	AVG	RESP	.00070	.07078	.06951	.08210	.07770	.06002
2	AVG	SP D	.00094	.07005	.06016	.09051	.11611	.13701
	AVG	RESP	.00171	.06240	.06262	.09000	.09494	.02700
3	AVG	SP D	.15767	.12493	.11002	.11908	.12142	.12702
	AVG	RESP	.01146	.02018	.03022	.02993	.02595	.01019
4	AVG	SP D	.092136	.03148	.00000	.07247	1.00055	2.00000
	AVG	RESP	.00097	.03034	.05097	.05002	.00095	.00741
5	AVG	SP D	.07060	.00363	.03280	.06807	.01162	.00702
	AVG	RESP	.07060	.06008	.06247	.05009	.00114	.00007
6	AVG	SP D	.00312	.13013	.10200	.02994	.00490	.07077
	AVG	RESP	.00090	.00929	.00746	.09397	.00827	.07904
ALL	AVG	SP D	1.02323	.23930	.26526	.35524	.52733	.72003
	AVG	RESP	.00020	.07701	.06537	.06143	.00000	.05127
UNIFORM PRECISION COMBINATION DESIGN:				6 RADIAL AND 5 INNER VERTICES R2/R1= .321				AVERAGE
SURFACE	ERR	ST DEV	0.00	0.03	0.06	0.09	0.12	0.15
1	AVG	SP D	.00018	.04096	.07267	.10874	.15548	.21907
	AVG	RESP	.00051	.06060	.07017	.09073	.09171	.04478
2	AVG	SP D	.00000	.05212	.07073	.09007	.11103	.13203
	AVG	RESP	.00030	.07739	.06037	.09028	.09071	.00005
3	AVG	SP D	.17609	.12034	.09060	.08007	.00027	.00005
	AVG	RESP	.00220	.07928	.09340	.09390	.09232	.07911
4	AVG	SP D	.11294	.17092	.52093	.09996	.07057	2.00003
	AVG	RESP	.02347	.00000	.01311	.02016	.00499	.00740
5	AVG	SP D	1.22010	.03713	1.00032	2.00001	1.00394	1.17005
	AVG	RESP	.05770	.05975	.06175	.06370	.00570	.00703
6	AVG	SP D	.02052	2.10027	.30070	.28954	.25923	.24004
	AVG	RESP	.00090	.05083	.09039	.09472	.09588	.09005
ALL	AVG	SP D	.27075	.08012	.37350	.72400	.30192	.75009
	AVG	RESP	.01050	.07044	.09055	.09492	.00223	.00006
UNIFORM PRECISION COMBINATION DESIGN:				6 RADIAL AND 5 INNER VERTICES R2/R1= .374				AVERAGE
SURFACE	ERR	ST DEV	0.00	0.03	0.06	0.09	0.12	0.15
1	AVG	SP D	.00054	.05477	.09063	.13853	.21133	.32095
	AVG	RESP	.00184	.08141	.09712	.09007	.05338	.72008
2	AVG	SP D	.00004	.05464	.07444	.09751	.12318	.15001
	AVG	RESP	.00070	.07287	.09055	.09005	.09580	.06010
3	AVG	SP D	.10019	.14086	.11209	.09007	.09028	.10104
	AVG	RESP	.05043	.07079	.09106	.09244	.09702	.07005
4	AVG	SP D	.11277	.19067	.53610	.02192	4.93914	11.00008
	AVG	RESP	.01001	.05954	.09134	.00100	.09981	.03906
5	AVG	SP D	.00090	.00060	1.03743	4.01633	1.11092	.92098
	AVG	RESP	.07084	.07026	.07569	.07814	.08062	.08001
6	AVG	SP D	.02044	1.00061	.32143	.26700	.24923	.24005
	AVG	RESP	.00090	.05083	.09066	.09041	.09085	.09704
ALL	AVG	SP D	.23133	.08052	.36184	1.00694	1.12034	2.13002
	AVG	RESP	.01780	.06095	.09087	.08759	.05268	.79701
UNIFORM PRECISION COMBINATION DESIGN:				6 RADIAL AND 7 INNER VERTICES R2/R1= .408				AVERAGE
SURFACE	ERR	ST DEV	0.00	0.03	0.06	0.09	0.12	0.15
1	AVG	SP D	.00025	.05050	.09074	.12024	.16395	.21705
	AVG	RESP	.00040	.08245	.09253	.09004	.09196	.05707
2	AVG	SP D	.00021	.05412	.07130	.09039	.10018	.12001
	AVG	RESP	.00032	.07009	.09207	.09019	.09014	.09093
3	AVG	SP D	.21075	.13098	.10060	.08753	.08703	.08001
	AVG	RESP	.09777	.07026	.09040	.09007	.09454	.09002
4	AVG	SP D	.11344	.13261	.20016	.09005	.77033	.92009
	AVG	RESP	.01274	.02912	.07042	.73741	.07717	.08006
5	AVG	SP D	.00152	.07068	2.05283	.91715	1.15978	1.19705
	AVG	RESP	.00172	.08009	.09056	.08014	.08779	.08001
6	AVG	SP D	.02038	2.10719	24.51565	.92008	.07007	.08746
	AVG	RESP	.00090	.07057	.09066	.09008	.09054	.09001
ALL	AVG	SP D	.23192	.08168	4.01824	.47302	.49450	.02008
	AVG	RESP	.01082	.08058	.09096	.06355	.05769	.02900
UNIFORM PRECISION COMBINATION DESIGN:				6 RADIAL AND 8 INNER VERTICES R2/R1= .431				AVERAGE
SURFACE	ERR	ST DEV	0.00	0.03	0.06	0.09	0.12	0.15
1	AVG	SP D	.00035	.05003	.07000	.10590	.14193	.18402
	AVG	RESP	.00017	.08007	.07006	.09001	.09379	.09000
2	AVG	SP D	.00051	.05008	.06051	.07009	.09091	.11000
	AVG	RESP	.00017	.07004	.06000	.09002	.09154	.09009
3	AVG	SP D	.20000	.16001	.11000	.09005	.08720	.08000
	AVG	RESP	.00000	.05042	.07000	.09705	.09307	.09000
4	AVG	SP D	.11003	.12005	.20000	.52147	1.22839	1.20005
	AVG	RESP	.00002	.02007	.09005	.77002	.07504	.09001
5	AVG	SP D	.00110	.06008	23.00074	.98172	.08048	1.30998
	AVG	RESP	.00120	.09017	.09033	.09004	.09383	.09000
6	AVG	SP D	.00035	.09073	5.40000	7.55339	1.44354	1.02707
	AVG	RESP	.00000	.02006	.07000	.41000	.72100	.09005
ALL	AVG	SP D	.23002	.08000	4.01650	1.25002	.09002	.00004
	AVG	RESP	.00039	.02003	.09000	.70000	.09009	.02008

UNIFORM PRECISION COMBINATION DESIGN: 6 RADIAL AND 10 INNER VERTICES R2/R1= .447									
SURFACE	ERR	ST DEV	0.00	0.03	0.06	0.09	0.12	0.15	AVERAGE
1	AVG	SP D	.05492	.05529	.07153	.10012	.13744	.18003	.10032
	AVG	PESP	.97713	.97716	.97716	.97716	.97716	.97716	.97716
2	AVG	SP D	.05160	.05160	.06221	.07094	.09244	.10041	.07390
	AVG	PESP	.97721	.97722	.97722	.97722	.97722	.97722	.97722
3	AVG	SP D	.05044	.05044	.06113	.06977	.08375	.08100	.06101
	AVG	PESP	.97775	.97775	.97775	.97775	.97775	.97775	.97775
4	AVG	SP D	.11580	.12050	.20369	.40840	4.05343	3.35300	1.51122
	AVG	PESP	.97752	.97752	.97752	.97752	.97752	.97752	.97752
5	AVG	SP D	.02949	.02949	.04403	.04875	.08762	1.71713	1.23526
	AVG	PESP	.60960	.60960	.70094	.70094	.70107	.70118	.70045
6	AVG	SP D	.02734	.02734	.03379	.03379	.03379	.03379	.03379
	AVG	PESP	.97703	.97703	.97703	.97703	.97703	.97703	.97703
ALL	AVG	SP D	.02949	.02949	.04403	.04875	.08762	1.71713	1.23526
	AVG	PESP	.60960	.60960	.70094	.70094	.70107	.70118	.70045
UNIFORM PRECISION COMBINATION DESIGN: 6 RADIAL AND 10 INNER VERTICES R2/R1= .459									
SURFACE	ERR	ST DEV	0.00	0.03	0.06	0.09	0.12	0.15	AVERAGE
1	AVG	SP D	.05409	.05409	.07325	.10209	.14554	.20001	.10733
	AVG	PESP	.97725	.97725	.97725	.97725	.97725	.97725	.97725
2	AVG	SP D	.05400	.05400	.06495	.06495	.10217	.12108	.06006
	AVG	PESP	.97741	.97741	.97741	.97741	.97741	.97741	.97741
3	AVG	SP D	.02171	.02171	.03115	.03115	.07633	.07446	.02405
	AVG	PESP	.67125	.67125	.67125	.67125	.67125	.67125	.67125
4	AVG	SP D	.11700	.11700	.25746	1.47376	1.49168	0.08092	2.00092
	AVG	PESP	.60047	.60047	.82294	.71226	.60087	.55212	.75462
5	AVG	SP D	.01780	.01780	.03673	.03673	.03673	.03673	.03673
	AVG	PESP	.70713	.70713	.70727	.70704	.70020	.70095	.70772
6	AVG	SP D	.02834	.02834	.030105	.030105	1.08006	.93714	1.92207
	AVG	PESP	.90090	.90090	.90090	.90090	.90090	.90090	.90090
ALL	AVG	SP D	.02834	.02834	.030105	.030105	1.08006	.93714	1.92207
	AVG	PESP	.90090	.90090	.90090	.90090	.90090	.90090	.90090
UNIFORM PRECISION COMBINATION DESIGN: 7 RADIAL AND 5 INNER VERTICES R2/R1= .256									
SURFACE	ERR	ST DEV	0.00	0.03	0.06	0.09	0.12	0.15	AVERAGE
1	AVG	SP D	.03699	.03699	.11595	.21045	.42203	2.91290	.62082
	AVG	PESP	.98552	.98552	.98552	.98552	.98552	.98552	.98552
2	AVG	SP D	.03600	.03600	.08231	.11028	.16961	.25178	.11904
	AVG	PESP	.98555	.98555	.98555	.98555	.98555	.98555	.98555
3	AVG	SP D	.18185	.18185	.24694	.16205	.10220	.09409	.20174
	AVG	PESP	.90293	.90293	.87177	.89041	.91845	.88536	.89100
4	AVG	SP D	.11670	.11670	.6.09451	.74524	4.57386	.84770	2.10508
	AVG	PESP	.92470	.92470	.77408	.63089	.43458	.49177	.67140
5	AVG	SP D	1.02257	1.02257	1.02304	1.02304	1.02372	1.02372	1.02327
	AVG	PESP	.64010	.64010	.64556	.64375	.64191	.64004	.64461
6	AVG	SP D	.02411	.02411	.17280	.17101	.17630	.18007	.14958
	AVG	PESP	.99098	.99098	.99032	.99914	.99090	.99079	.99927
ALL	AVG	SP D	.02411	.02411	.17280	.17101	.17630	.18007	.14958
	AVG	PESP	.99098	.99098	.99032	.99914	.99090	.99079	.99927
UNIFORM PRECISION COMBINATION DESIGN: 7 RADIAL AND 6 INNER VERTICES R2/R1= .330									
SURFACE	ERR	ST DEV	0.00	0.03	0.06	0.09	0.12	0.15	AVERAGE
1	AVG	SP D	.03040	.03040	.09058	.15442	.23330	.36009	.13788
	AVG	PESP	.99305	.99305	.99444	.98557	.70419	.67001	.87482
2	AVG	SP D	.03013	.03013	.07696	.10332	.13480	.17115	.09718
	AVG	PESP	.99300	.99300	.99251	.90392	.85141	.78700	.90015
3	AVG	SP D	.34173	.34173	.08763	.07800	.07855	.08098	.13028
	AVG	PESP	.85422	.85422	.92143	.95101	.92455	.88001	.91785
4	AVG	SP D	.11369	.11369	.50660	1.44803	2.14328	20.17774	5.50039
	AVG	PESP	.92217	.92217	.76663	.71306	.51380	.36143	.60942
5	AVG	SP D	1.13769	1.13769	32.23345	1.57308	1.14914	1.03070	6.45815
	AVG	PESP	.60000	.60000	.65000	.65401	.65111	.64011	.65527
6	AVG	SP D	.02002	.02002	.23023	.22570	.22493	.22015	.19065
	AVG	PESP	.99090	.99090	.99065	.99031	.99030	.99025	.99073
ALL	AVG	SP D	.02002	.02002	.23023	.22570	.22493	.22015	.19065
	AVG	PESP	.99090	.99090	.99065	.99031	.99030	.99025	.99073
UNIFORM PRECISION COMBINATION DESIGN: 7 RADIAL AND 7 INNER VERTICES R2/R1= .374									
SURFACE	ERR	ST DEV	0.00	0.03	0.06	0.09	0.12	0.15	AVERAGE
1	AVG	SP D	.04014	.04014	.09629	.14015	.21750	.32096	.14007
	AVG	PESP	.97779	.97779	.97779	.97779	.97779	.97779	.97779
2	AVG	SP D	.04195	.04195	.07467	.09073	.12794	.10404	.09372
	AVG	PESP	.97780	.97780	.97780	.97780	.97780	.97780	.97780
3	AVG	SP D	.04050	.04050	.10779	.09277	.08990	.09247	.10118
	AVG	PESP	.97770	.97770	.97770	.97770	.97770	.97770	.97770
4	AVG	SP D	.11300	.11300	.56220	1.19004	7.78997	5.71723	2.01852
	AVG	PESP	.97750	.97750	.97750	.97750	.97750	.97750	.97750
5	AVG	SP D	1.50733	1.50733	2.05743	1.05711	1.24971	1.00009	2.11083
	AVG	PESP	.67309	.67309	.66055	.66310	.65980	.65012	.66075
6	AVG	SP D	.02797	.02797	.22515	.21900	.21042	.21002	.19418
	AVG	PESP	.99099	.99099	.99070	.99007	.99055	.99043	.99085
ALL	AVG	SP D	.02797	.02797	.22515	.21900	.21042	.21002	.19418
	AVG	PESP	.99099	.99099	.99070	.99007	.99055	.99043	.99085

UNIFORM PRECISION COMBINATION DESIGN: 7 RADIAL AND 8 INNER VERTICES R2/R1= .404									
SURFACE	ERR	ST DEV	0.00	0.03	0.06	0.09	0.12	0.15	AVERAGE
1	AVG SP D	.04405	.06106	.09251	.13350	.20413	.30095	.14087	
	AVG PESP	.07473	.07063	.09476	.09120	.02276	.71608	.00983	
2	AVG SP D	.04501	.05517	.07259	.09448	.12620	.15124	.08980	
	AVG PESP	.07474	.06524	.09346	.09119	.06906	.81021	.01457	
3	AVG SP D	.07064	.17780	.11668	.10123	.09670	.09706	.21503	
	AVG PESP	.02218	.08570	.09459	.09435	.02211	.09521	.00134	
4	AVG SP D	.11314	.27409	.79427	1.13734	1.00107	2.29030	.93604	
	AVG PESP	.91327	.78241	.73674	.72148	.54245	.39066	.60120	
5	AVG SP D	3.05310	2.06019	1.27194	1.04023	.96467	.94090	1.09036	
	AVG PESP	.60772	.67938	.67596	.67245	.60087	.66521	.67410	
6	AVG SP D	.02794	.10453	.25022	.23790	.23397	.23109	.21429	
	AVG PESP	.99999	.99703	.99828	.99815	.99799	.99703	.99836	
ALL	AVG SP D	.76416	.82214	.43304	.45928	.43673	.67102	.54773	
	AVG PESP	.89504	.80022	.87409	.85760	.80400	.74707	.84324	
UNIFORM PRECISION COMBINATION DESIGN: 7 RADIAL AND 9 INNER VERTICES R2/R1= .425									
SURFACE	ERR	ST DEV	0.00	0.03	0.06	0.09	0.12	0.15	AVERAGE
1	AVG SP D	.04790	.06535	.09582	.14058	.22450	.35012	.15572	
	AVG PESP	.07783	.06929	.09451	.09417	.02291	.68037	.07816	
2	AVG SP D	.04511	.05702	.07566	.09373	.12630	.16014	.09434	
	AVG PESP	.07683	.06292	.09025	.09078	.06252	.60094	.09944	
3	AVG SP D	1.76160	.18023	.12200	.10652	.10174	.10108	.39561	
	AVG PESP	.78409	.27950	.93246	.93249	.91740	.69428	.69070	
4	AVG SP D	.11371	.31968	.90879	.65438	2.62480	4.67004	1.61671	
	AVG PESP	.99929	.74432	.73050	.68211	.53390	.40936	.60636	
5	AVG SP D	3.20004	1.22275	.97400	.90359	.90911	.99023	1.30745	
	AVG PESP	.69120	.68798	.68460	.68114	.67761	.67400	.66277	
6	AVG SP D	.02793	.10060	.23457	.22642	.22192	.22022	.20011	
	AVG PESP	.99999	.99755	.99795	.99777	.99755	.99736	.99803	
ALL	AVG SP D	.86656	.75727	.40264	.35604	.73470	1.11009	.63933	
	AVG PESP	.89039	.87368	.87186	.84914	.79865	.74575	.83791	
UNIFORM PRECISION COMBINATION DESIGN: 7 RADIAL AND 10 INNER VERTICES R2/R1= .440									
SURFACE	ERR	ST DEV	0.00	0.03	0.06	0.09	0.12	0.15	AVERAGE
1	AVG SP D	.05078	.06927	.10692	.17327	.29580	.59498	.21517	
	AVG PESP	.07508	.06509	.09365	.06294	.72381	.53510	.63245	
2	AVG SP D	.05103	.06100	.08112	.10764	.14219	.18708	.10514	
	AVG PESP	.07508	.05038	.09308	.08920	.63044	.75423	.60971	
3	AVG SP D	2.13051	.23057	.15423	.13074	.12318	.12144	.40311	
	AVG PESP	.75081	.24278	.89216	.90221	.39144	.67036	.60683	
4	AVG SP D	.11457	.69173	1.25370	.74854	2.94627	.91302	1.11141	
	AVG PESP	.99561	.68082	.71453	.63215	.49004	.53090	.60101	
5	AVG SP D	1.55393	1.08303	.93683	.68230	.67548	.91124	1.04130	
	AVG PESP	.69394	.69597	.69272	.68931	.68623	.68208	.69103	
6	AVG SP D	.02792	.23152	.19500	.18849	.18745	.18001	.10973	
	AVG PESP	.99999	.99058	.99860	.99836	.99811	.99709	.99859	
ALL	AVG SP D	.65562	.79085	.45463	.37166	.76173	.48016	.52098	
	AVG PESP	.88675	.85492	.86059	.82907	.77001	.72926	.82160	
UNIFORM PRECISION COMBINATION DESIGN: 8 RADIAL AND 5 INNER VERTICES R2/R1= .170									
SURFACE	ERR	ST DEV	0.00	0.03	0.06	0.09	0.12	0.15	AVERAGE
1	AVG SP D	.03524	.05414	.08521	.12917	.19405	.30108	.13320	
	AVG PESP	.03733	.07193	.03972	.88079	.78873	.07332	.87308	
2	AVG SP D	.03530	.05293	.07807	.11270	.16219	.23915	.11340	
	AVG PESP	.08731	.07123	.09000	.08096	.80755	.70054	.80361	
3	AVG SP D	.05567	.05012	.06500	.08335	.11183	.15908	.06078	
	AVG PESP	.06067	.07154	.09675	.09516	.91313	.83027	.93532	
4	AVG SP D	.11227	1.14179	.45081	.48030	1.25523	2.97140	1.07147	
	AVG PESP	.93372	.77366	.77095	.74146	.63873	.49208	.72502	
5	AVG SP D	1.02381	1.05304	1.19675	1.04400	21.06475	1.04071	4.00943	
	AVG PESP	.63093	.63466	.62030	.62411	.61683	.61034	.62074	
6	AVG SP D	.02421	.18195	.17056	.16938	.17072	.17313	.14899	
	AVG PESP	.99098	.99940	.99045	.99942	.99937	.99929	.99948	
ALL	AVG SP D	.21459	.42348	.34103	.43702	.309313	.91011	1.03754	
	AVG PESP	.91049	.88708	.87452	.84742	.79439	.72096	.84004	
UNIFORM PRECISION COMBINATION DESIGN: 9 RADIAL AND 6 INNER VERTICES R2/R1= .280									
SURFACE	ERR	ST DEV	0.00	0.03	0.06	0.09	0.12	0.15	AVERAGE
1	AVG SP D	.03724	.05760	.08899	.13376	.20128	.31036	.13924	
	AVG PESP	.04002	.06748	.03207	.06838	.77209	.05027	.80297	
2	AVG SP D	.03734	.05722	.08507	.12551	.18310	.27331	.12747	
	AVG PESP	.09100	.06681	.03184	.07237	.78497	.07936	.67006	
3	AVG SP D	.04091	.04372	.06276	.08379	.13398	.21102	.09883	
	AVG PESP	.03343	.08309	.07654	.95520	.90510	.01002	.93551	
4	AVG SP D	.17059	.56957	.40327	.48030	.05204	4.00002	1.19502	
	AVG PESP	.92067	.78486	.75061	.72202	.61053	.40090	.71120	
5	AVG SP D	1.04117	.08902	1.03502	1.22659	2.04045	2.10490	1.45752	
	AVG PESP	.65064	.64085	.64105	.63525	.62944	.62002	.63814	
6	AVG SP D	.02410	.17941	.16680	.16370	.16303	.16036	.14408	
	AVG PESP	.99098	.99941	.99048	.99949	.99946	.99946	.99955	
ALL	AVG SP D	.21473	.71602	.32011	.37010	.04075	1.20000	.52703	
	AVG PESP	.90229	.80142	.87193	.84231	.70401	.70007	.83024	

UNIFORM PRECISION COMBINATION DESIGN: 8 RADIAL AND 7 INNER VERTICES R2/R1= .337									
SURFACE/ERR	ST DEV	0.00	0.03	0.06	0.09	0.12	0.15	AVERAGE	
1	AVG CP D	.00000	.00074	.00236	.00776	.01735	.02659	.13076	
	AVG PESP	.00000	.00039	.00152	.00452	.00960	.01513	.07360	
2	AVG CP D	.00000	.00057	.00160	.00460	.00971	.01426	.11025	
	AVG PESP	.00000	.00026	.00070	.00237	.00498	.00740	.00295	
3	AVG CP D	.00000	.00055	.00155	.00455	.00967	.01419	.00955	
	AVG PESP	.00000	.00028	.00073	.00237	.00498	.00740	.00295	
4	AVG CP D	.00000	.00044	.00114	.00314	.00620	.00926	.00979	
	AVG PESP	.00000	.00024	.00065	.00192	.00381	.00570	.00714	
5	AVG CP D	1.16499	1.00593	.96599	.98709	1.07769	1.32416	1.00774	
	AVG PESP	.00000	.00055	.00155	.00455	.00967	.01419	.00955	
6	AVG CP D	.00000	.00022	.00062	.00162	.00312	.00462	.00550	
	AVG PESP	.00000	.00010	.00034	.00084	.00168	.00252	.00336	
ALL	AVG CP D	.25557	.00236	.00698	.03179	.01857	.00731	.42993	
	AVG PESP	.00000	.00022	.00062	.00162	.00312	.00462	.00550	
UNIFORM PRECISION COMBINATION DESIGN: 8 RADIAL AND 8 INNER VERTICES R2/R1= .374									
SURFACE/ERR	ST DEV	0.00	0.03	0.06	0.09	0.12	0.15	AVERAGE	
1	AVG CP D	.00190	.00125	.00620	.01324	.01907	.03005	.13023	
	AVG PESP	.00000	.00023	.00082	.00182	.00339	.00509	.00557	
2	AVG CP D	.00194	.00066	.00102	.00304	.00620	.00926	.11546	
	AVG PESP	.00000	.00025	.00076	.00182	.00339	.00509	.00557	
3	AVG CP D	.00100	.00034	.00086	.00170	.00311	.00461	.00539	
	AVG PESP	.00000	.00016	.00056	.00111	.00212	.00309	.00391	
4	AVG CP D	.00000	.00040	.00115	.00315	.00620	.00926	.00700	
	AVG PESP	.00000	.00014	.00039	.00092	.00182	.00270	.00316	
5	AVG CP D	1.50322	1.11270	.90172	.95091	.95284	.99107	1.00381	
	AVG PESP	.00000	.00040	.00115	.00315	.00620	.00926	.00700	
6	AVG CP D	.00290	.00446	.00514	.00696	.00834	.00959	.00804	
	AVG PESP	.00000	.00035	.00091	.00192	.00339	.00509	.00557	
ALL	AVG CP D	.20242	.00080	.00276	.00559	.01103	.00799	.40799	
	AVG PESP	.00000	.00013	.00052	.00111	.00212	.00309	.00381	
UNIFORM PRECISION COMBINATION DESIGN: 8 RADIAL AND 9 INNER VERTICES R2/R1= .401									
SURFACE/ERR	ST DEV	0.00	0.03	0.06	0.09	0.12	0.15	AVERAGE	
1	AVG CP D	.00450	.00755	.01224	.01558	.02478	.04449	.17704	
	AVG PESP	.00000	.00080	.00182	.00321	.00521	.00626	.00864	
2	AVG CP D	.00451	.00005	.00089	.00187	.00258	.00317	.00588	
	AVG PESP	.00000	.00011	.00029	.00040	.00062	.00075	.00076	
3	AVG CP D	.00231	.00067	.00054	.00130	.00173	.00260	.00254	
	AVG PESP	.00000	.00008	.00012	.00031	.00067	.00111	.00146	
4	AVG CP D	.00014	1.10456	.49729	.00893	.00653	.00370	2.05030	
	AVG PESP	.00000	.00014	.00040	.00092	.00182	.00270	.00316	
5	AVG CP D	2.97000	1.19113	.97458	.91900	.93177	1.01249	1.33478	
	AVG PESP	.00000	.00058	.00126	.00205	.00305	.00307	.00331	
6	AVG CP D	.00290	.00443	.00457	.00694	.00838	.00955	.00874	
	AVG PESP	.00000	.00046	.00092	.00182	.00339	.00509	.00557	
ALL	AVG CP D	.53402	.00050	.00164	.00355	.00620	.00799	.00205	
	AVG PESP	.00000	.00019	.00052	.00111	.00212	.00309	.00381	
UNIFORM PRECISION COMBINATION DESIGN: 8 RADIAL AND 10 INNER VERTICES R2/R1= .420									
SURFACE/ERR	ST DEV	0.00	0.03	0.06	0.09	0.12	0.15	AVERAGE	
1	AVG CP D	.00474	.00670	.01241	.01554	.02462	.04319	.17597	
	AVG PESP	.00000	.00074	.00182	.00321	.00521	.00626	.00864	
2	AVG CP D	.00473	.00074	.00099	.00170	.00198	.00273	.00254	
	AVG PESP	.00000	.00004	.00011	.00029	.00062	.00111	.00125	
3	AVG CP D	.00200	.00032	.00028	.00145	.00158	.00203	.00259	
	AVG PESP	.00000	.00006	.00010	.00021	.00033	.00043	.00041	
4	AVG CP D	.00057	.00252	.00109	.00297	.00619	.00423	2.24708	
	AVG PESP	.00000	.00005	.00013	.00038	.00084	.00108	.00097	
5	AVG CP D	5.27343	1.08132	1.18146	.99822	.92732	.90412	1.00096	
	AVG PESP	.00000	.00031	.00074	.00145	.00252	.00309	.00309	
6	AVG CP D	.00290	.00349	.00552	.00628	.00712	.00813	.00802	
	AVG PESP	.00000	.00049	.00092	.00182	.00339	.00509	.00557	
ALL	AVG CP D	.92471	.00200	.00323	.00714	.00409	.00309	.77461	
	AVG PESP	.00000	.00022	.00052	.00111	.00212	.00309	.00381	
UNIFORM PRECISION COMBINATION DESIGN: 8 RADIAL AND 11 INNER VERTICES R2/R1= .217									
SURFACE/ERR	ST DEV	0.00	0.03	0.06	0.09	0.12	0.15	AVERAGE	
1	AVG CP D	.00660	.00121	.00246	.00444	.00764	.01879	.00788	
	AVG PESP	.00000	.00051	.00152	.00452	.00960	.01513	.00695	
2	AVG CP D	.00405	.00023	.00037	.00133	.00206	.00306	.00295	
	AVG PESP	.00000	.00006	.00010	.00020	.00040	.00060	.00062	
3	AVG CP D	.00007	.00026	.00060	.00150	.00249	.00358	.00384	
	AVG PESP	.00000	.00009	.00020	.00040	.00080	.00120	.00160	
4	AVG CP D	.00000	.00076	.00173	.00334	.00653	.00926	.00203	
	AVG PESP	.00000	.00013	.00034	.00084	.00168	.00252	.00336	
5	AVG CP D	1.01700	1.00265	2.04658	1.35231	1.27267	27.23002	5.70971	
	AVG PESP	.00000	.00015	.00038	.00092	.00182	.00270	.00316	
6	AVG CP D	.00017	.00010	.00083	.00282	.00318	.00401	.00260	
	AVG PESP	.00000	.00004	.00010	.00020	.00040	.00060	.00062	
ALL	AVG CP D	.21139	.00087	.00043	.00432	.00209	.00209	1.22904	
	AVG PESP	.00000	.00010	.00020	.00040	.00080	.00120	.00160	

UNIFORM PRECISION COMBINATION DESIGN:		0 RADIAL AND 7 INNER VERTICES R2/K12		.295				
SURFACE	ST DEV	0.00	0.03	0.06	0.09	0.12	0.15	AVERAGE
1	AVG SP D	.04780	.05355	.07420	.10054	.13428	.17940	.07665
	AVG RESP	.96051	.07014	.98602	.99051	.05423	.78204	.90767
2	AVG SP D	.03794	.05904	.09692	.15024	.20888	.33405	.19250
	AVG RESP	.93451	.07312	.94661	.38036	.77571	.59701	.80099
3	AVG SP D	.04772	.08045	.65685	.05105	1.09942	.07400	.63168
	AVG RESP	.90093	.07977	.85444	.77271	.76070	.74537	.64965
4	AVG SP D	.10733	.14298	.21418	.90091	.08051	.72902	.49782
	AVG RESP	.92009	.07967	.78709	.07908	.01084	.00011	.74893
5	AVG SP D	1.05081	.08799	1.22469	4.15191	1.21350	1.36098	1.60918
	AVG RESP	.65519	.64957	.64390	.02319	.03241	.02007	.64097
6	AVG SP D	.02008	.21747	.21671	.25479	.35390	.73007	.30028
	AVG RESP	.99098	.09903	.99055	.99024	.98486	.03075	.90827
ALL	AVG SP D	.21061	.25791	.41394	1.02506	.75877	.70232	.50302
	AVG RESP	.92330	.00855	.86277	.81402	.77113	.09078	.82942
UNIFORM PRECISION COMBINATION DESIGN:		0 RADIAL AND 9 INNER VERTICES R2/K12		.342				
SURFACE	ST DEV	0.00	0.03	0.06	0.09	0.12	0.15	AVERAGE
1	AVG SP D	.03083	.05976	.08651	.12149	.10908	.23070	.11923
	AVG RESP	.99205	.66444	.93269	.32119	.00410	.70239	.81786
2	AVG SP D	.03682	.06041	.11437	.20170	.40334	3.97102	.79944
	AVG RESP	.99204	.66764	.93675	.04234	.05721	.37113	.79205
3	AVG SP D	.04261	.08374	.67052	.08035	1.04550	.94003	.57009
	AVG RESP	.99162	.08339	.81854	.72738	.74702	.71003	.83066
4	AVG SP D	.10652	.15098	.37669	.97544	.78198	1.06001	.70974
	AVG RESP	.92474	.05018	.73065	.01895	.09457	.56003	.71548
5	AVG SP D	1.13610	.05099	1.13735	3.10815	1.14740	1.75073	1.54867
	AVG RESP	.65464	.65940	.65418	.64896	.04374	.03001	.60157
6	AVG SP D	.02002	.19440	.23077	.44490	4.49454	1.22196	1.10393
	AVG RESP	.99098	.09904	.99003	.99047	.03028	.86025	.90901
ALL	AVG SP D	.24083	.25255	.43754	.92009	1.34032	1.06008	.80962
	AVG RESP	.92440	.00435	.94391	.77908	.07949	.04422	.79611
UNIFORM PRECISION COMBINATION DESIGN:		0 RADIAL AND 10 INNER VERTICES R2/K12		.374				
SURFACE	ST DEV	0.00	0.03	0.06	0.09	0.12	0.15	AVERAGE
1	AVG SP D	.04197	.05997	.08249	.11082	.14738	.19002	.10656
	AVG RESP	.99091	.06095	.93624	.89359	.83190	.75014	.89279
2	AVG SP D	.04190	.06031	.10562	.17171	.29059	.03000	.21920
	AVG RESP	.99091	.06099	.93607	.86962	.73903	.52096	.83660
3	AVG SP D	.03852	.07127	.28619	.55449	.96117	1.76297	.61244
	AVG RESP	.99399	.08017	.86296	.77152	.59015	.06471	.82862
4	AVG SP D	.10627	.14095	.24511	3.35342	.50938	.00105	.82711
	AVG RESP	.92110	.66377	.76017	.65746	.59459	.57162	.72816
5	AVG SP D	1.50412	.05092	.96385	1.45332	1.59617	1.12048	1.20604
	AVG RESP	.67307	.66018	.66234	.05301	.05371	.04093	.60096
6	AVG SP D	.02793	.19117	.20122	.25359	.38080	.95739	.33037
	AVG RESP	.99099	.09922	.99041	.99430	.97515	.70094	.94475
ALL	AVG SP D	.29347	.24066	.31408	.98237	.04960	.87902	.50139
	AVG RESP	.92500	.00038	.85953	.80758	.74742	.64095	.81531
UNIFORM PRECISION COMBINATION DESIGN:		0 RADIAL AND 10 INNER VERTICES R2/K12		.398				
SURFACE	ST DEV	0.00	0.03	0.06	0.09	0.12	0.15	AVERAGE
1	AVG SP D	.04422	.05771	.07399	.09305	.11773	.14797	.08921
	AVG RESP	.97029	.06487	.94299	.91149	.86035	.81205	.91327
2	AVG SP D	.04422	.06259	.09175	.13481	.19990	.30012	.14024
	AVG RESP	.97029	.06756	.94424	.90110	.82713	.71031	.88961
3	AVG SP D	.03477	.06157	.13818	.57209	.98762	1.00432	.50053
	AVG RESP	.99481	.09014	.95044	.78929	.07501	.52092	.82010
4	AVG SP D	.10637	.13337	.17075	.27328	3.45038	.04075	.79915
	AVG RESP	.91709	.07720	.81008	.73050	.00418	.01494	.70910
5	AVG SP D	2.62010	1.04156	.01534	.96038	1.28793	3.04092	1.77870
	AVG RESP	.69006	.67000	.67096	.06014	.00135	.05007	.00858
6	AVG SP D	.02790	.20270	.18202	.17808	.17780	.17925	.10798
	AVG RESP	.99099	.09925	.99030	.99340	.99938	.99904	.99946
ALL	AVG SP D	.47001	.25902	.26334	.36382	1.03791	1.12222	.50864
	AVG RESP	.92629	.01217	.88635	.83299	.78257	.72045	.84335
UNIFORM PRECISION COMBINATION DESIGN:		10 RADIAL AND 0 INNER VERTICES R2/K12		.129				
SURFACE	ST DEV	0.00	0.03	0.06	0.09	0.12	0.15	AVERAGE
1	AVG SP D	.03404	.04946	.07690	.11335	.16340	.23420	.11214
	AVG RESP	.98778	.08149	.96425	.93090	.87412	.78001	.92120
2	AVG SP D	.03405	.04429	.07061	.14111	.20277	.00001	.19407
	AVG RESP	.99779	.08936	.97884	.94009	.83017	.55009	.88011
3	AVG SP D	.05001	.09359	.63056	.28383	.35010	.37708	.33470
	AVG RESP	.97070	.06031	.83470	.85323	.00100	.01004	.81983
4	AVG SP D	.11120	.10340	.12219	.15329	.19394	.24002	.15548
	AVG RESP	.93034	.02461	.89235	.84810	.79482	.73001	.80495
5	AVG SP D	1.02050	1.15873	2.34620	1.00548	1.19491	1.30018	1.44051
	AVG RESP	.63713	.63314	.62896	.02457	.01997	.01010	.02049
6	AVG SP D	.02025	.25040	.08091	.30785	.20072	.24074	.34921
	AVG RESP	.99098	.02018	.98020	.99131	.99354	.99447	.98095
ALL	AVG SP D	.21070	.26001	.00771	.43424	.40032	.03070	.43103
	AVG RESP	.91000	.00252	.87000	.86404	.82571	.75071	.80720

UNIFORM PRECISION COMBINATION DESIGN: 10 RADIAL AND 7 INNER VERTICES R2/R1 = .240								
SURFACE	ST DEV	0.00	0.03	0.06	0.09	0.12	0.15	AVERAGE
1	AVG SP D	.03550	.05426	.08829	.13735	.21155	.34201	.14509
	AVG PESP	.98590	.07079	.95233	.90134	.81069	.67334	.80385
2	AVG SP D	.03550	.05104	.09584	.19338	.46437	77.43072	13.04527
	AVG PESP	.98590	.08568	.96793	.89728	.84666	.21333	.70381
3	AVG SP D	.05139	.07011	1.27007	1.16285	5.20394	.34313	1.35174
	AVG PESP	.98169	.07506	.83792	.83924	.83540	.62326	.80325
4	AVG SP D	.10869	.10827	.13197	.17139	.23144	.32002	.17985
	AVG PESP	.93125	.01252	.87069	.81233	.74179	.66223	.82183
5	AVG SP D	1.02195	1.04527	1.67355	1.56521	1.14782	1.52001	1.32780
	AVG PESP	.04779	.64420	.64042	.63642	.63221	.62178	.63814
6	AVG SP D	.02814	10.52711	.30014	.25966	.23044	.22508	1.93623
	AVG PESP	.99896	.64031	.90112	.99406	.99563	.99007	.93763
ALL	AVG SP D	.21389	1.07701	.60013	.58034	1.24920	13.36405	2.99766
	AVG PESP	.92209	.85723	.87674	.84700	.77700	.66008	.82475
UNIFORM PRECISION COMBINATION DESIGN: 10 RADIAL AND 4 INNER VERTICES R2/R1 = .306								
SURFACE	ST DEV	0.00	0.03	0.06	0.09	0.12	0.15	AVERAGE
1	AVG SP D	.03623	.05591	.08420	.12032	.18574	.27736	.12773
	AVG PESP	.98413	.07088	.95494	.91354	.84155	.73029	.90059
2	AVG SP D	.03623	.05106	.09003	.16044	.34128	1.36134	.34126
	AVG PESP	.98413	.08504	.97139	.92005	.75585	.30044	.61948
3	AVG SP D	.04584	.06295	.71481	.61432	.34290	.41139	.39881
	AVG PESP	.98226	.08316	.81321	.64076	.61941	.60306	.67011
4	AVG SP D	.10736	.10591	.12789	.16364	.21245	.28206	.16663
	AVG PESP	.92775	.01088	.87350	.82301	.76283	.69373	.83228
5	AVG SP D	1.07660	.97730	1.12994	3.20335	1.32938	1.11016	1.40862
	AVG PESP	.65717	.65093	.65053	.64097	.64324	.63303	.64653
6	AVG SP D	.02807	1.77337	.39085	.27506	.24489	.23306	.49063
	AVG PESP	.99996	.66083	.98790	.99373	.99510	.99570	.93887
ALL	AVG SP D	.22239	.50408	.41929	.75318	.47612	.61003	.49895
	AVG PESP	.92357	.86199	.87524	.85034	.80300	.69072	.83598
UNIFORM PRECISION COMBINATION DESIGN: 10 RADIAL AND 9 INNER VERTICES R2/R1 = .346								
SURFACE	ST DEV	0.00	0.03	0.06	0.09	0.12	0.15	AVERAGE
1	AVG SP D	.04003	.05159	.07487	.10519	.14248	.18999	.10069
	AVG PESP	.98247	.07087	.96211	.93596	.89479	.83305	.93121
2	AVG SP D	.04003	.04647	.07473	.12236	.20333	.36105	.14176
	AVG PESP	.98247	.08544	.97806	.95107	.88548	.73394	.91954
3	AVG SP D	.04137	.05335	.12566	.88165	.75016	2.02220	.64574
	AVG PESP	.97211	.09016	.96666	.74202	.75474	.74306	.80489
4	AVG SP D	.10674	.10137	.11967	.14078	.17918	.21001	.14534
	AVG PESP	.92431	.01307	.89477	.84438	.80046	.76136	.85039
5	AVG SP D	1.20887	.98450	.96523	1.08090	1.33686	1.93006	1.33541
	AVG PESP	.66553	.66230	.65895	.65348	.65189	.64016	.65705
6	AVG SP D	.02802	.05085	.70496	.34417	.28357	.25302	.34515
	AVG PESP	.99096	.68027	.96262	.98323	.99266	.99335	.93045
ALL	AVG SP D	.24417	.28169	.34419	.44704	.50594	.63026	.45235
	AVG PESP	.92448	.01002	.90219	.85382	.83101	.78002	.80926
UNIFORM PRECISION COMBINATION DESIGN: 10 RADIAL AND 10 INNER VERTICES R2/R1 = .374								
SURFACE	ST DEV	0.00	0.03	0.06	0.09	0.12	0.15	AVERAGE
1	AVG SP D	.04190	.05155	.07269	.09377	.12875	.16400	.09295
	AVG PESP	.98292	.07017	.96332	.94180	.90994	.86037	.93968
2	AVG SP D	.04190	.04659	.07168	.11121	.17193	.27135	.11945
	AVG PESP	.98092	.08490	.97902	.95330	.91112	.81332	.93020
3	AVG SP D	.03770	.04074	.10446	.37225	1.03927	.80005	.40151
	AVG PESP	.98416	.09282	.97898	.79236	.67059	.67332	.85040
4	AVG SP D	.10549	.09962	.11746	.14105	.16669	.19079	.13879
	AVG PESP	.92101	.01183	.89677	.85571	.82094	.78427	.80342
5	AVG SP D	1.50498	1.03023	.98633	.96495	1.11303	1.42001	1.24926
	AVG PESP	.67307	.66960	.66604	.66230	.65860	.65472	.66407
6	AVG SP D	.02799	.06498	1.62483	.41501	.32190	.28731	.50702
	AVG PESP	.99099	.08068	.83072	.98423	.99019	.99222	.90434
ALL	AVG SP D	.20355	.07528	.40057	.35004	.49061	.60334	.41810
	AVG PESP	.92501	.92067	.89414	.86379	.82090	.79107	.87003
ORTHOGONAL COMBINATION DESIGN: 5 RADIAL AND 6 INNER VERTICES R2/R1 = .204								
SURFACE	ST DEV	0.00	0.03	0.06	0.09	0.12	0.15	AVERAGE
1	AVG SP D	.04097	.05078	.06964	.08941	.09746	.11005	.07039
	AVG PESP	.96240	.08256	.08751	.98537	.97576	.95300	.97579
2	AVG SP D	.04224	.06152	.07232	.09030	.11585	.14033	.09185
	AVG PESP	.97159	.07307	.96400	.94009	.92040	.88446	.94338
3	AVG SP D	.10402	.10101	.10554	.11906	.22271	.14000	.13221
	AVG PESP	.98300	.05143	.94607	.93345	.87980	.90035	.92848
4	AVG SP D	.74723	.40725	.24835	.25974	.33523	.27029	.37802
	AVG PESP	.77363	.82350	.85082	.87094	.89246	.88490	.88097
5	AVG SP D	.30538	.00169	.42009	.44109	.40537	.49394	.43459
	AVG PESP	.70061	.70138	.69250	.68323	.67329	.66002	.68716
6	AVG SP D	.03124	.16138	.45371	1.54768	1.10167	.42020	.62035
	AVG PESP	.00000	.00906	.06560	.65039	.75264	.90115	.92407
ALL	AVG SP D	.23305	.12076	.22661	.42125	.38005	.26003	.26890
	AVG PESP	.89570	.90017	.90120	.88120	.84900	.87137	.80497

ORTHOGONAL COMBINATION DESIGN: 5 RADIAL AND 7 INNER VERTICES R2/R1= .200									
SURFACE	ERR	ST DEV	0.00	0.03	0.06	0.09	0.12	0.15	AVERAGE
1	AVG	SP D	.06364	.06505	.07153	.09002	.12013	.19003	.10216
	AVG	PESP	.06478	.07010	.09160	.09352	.094723	.08126	.09447
2	AVG	SP D	.06554	.06687	.07190	.11390	.15390	.21203	.11077
	AVG	PESP	.06770	.06549	.09077	.09100	.07414	.08009	.091598
3	AVG	SP D	.14380	.11466	.10658	.10946	.11484	.12096	.11923
	AVG	PESP	.02154	.04063	.04196	.03460	.02207	.04038	.02753
4	AVG	SP D	1.03768	.00331	.27129	.33782	.26565	.21903	.40293
	AVG	PESP	.76384	.00953	.83776	.05546	.07513	.09017	.63865
5	AVG	SP D	.36076	.00243	.41770	.43500	.45460	.47720	.42929
	AVG	PESP	.70673	.09940	.69171	.08309	.07539	.06073	.60731
6	AVG	SP D	.03120	.19192	1.18481	2.36515	.46043	.33700	.70189
	AVG	PESP	.99698	.09030	.09011	.58570	.90914	.98411	.90289
ALL	AVG	SP D	.20949	.18004	.35663	.07526	.26260	.26026	.32205
	AVG	PESP	.89747	.09057	.02215	.02551	.07052	.05709	.07114
ORTHOGONAL COMBINATION DESIGN: 5 RADIAL AND 9 INNER VERTICES R2/R1= .304									
SURFACE	ERR	ST DEV	0.00	0.03	0.06	0.09	0.12	0.15	AVERAGE
1	AVG	SP D	.07249	.05948	.05714	.06507	.08730	.12444	.07765
	AVG	PESP	.06159	.08167	.09864	.09341	.07287	.05009	.07343
2	AVG	SP D	.06553	.06091	.09455	.10439	.12759	.15416	.10135
	AVG	PESP	.06479	.06037	.05674	.09373	.091077	.08005	.093884
3	AVG	SP D	.15388	.15223	.11158	.11212	.12739	.09102	.19497
	AVG	PESP	.02258	.01081	.03479	.02754	.00788	.07107	.091349
4	AVG	SP D	1.47564	.02076	.33581	.34341	.30859	.42005	.55314
	AVG	PESP	.75548	.00139	.02740	.03008	.05715	.05002	.02219
5	AVG	SP D	.39273	.01259	.43552	.46237	.49442	.53003	.45521
	AVG	PESP	.70403	.69420	.68388	.07019	.06235	.05107	.67822
6	AVG	SP D	.03130	.14079	.25957	.53736	5.94251	5.01042	2.12182
	AVG	PESP	.99990	.09909	.09526	.07941	.70001	.51706	.87627
ALL	AVG	SP D	.36576	.21146	.21403	.27078	1.18130	1.26002	.50403
	AVG	PESP	.88474	.09026	.09779	.09006	.04717	.08003	.80707
ORTHOGONAL COMBINATION DESIGN: 5 RADIAL AND 9 INNER VERTICES R2/R1= .330									
SURFACE	ERR	ST DEV	0.00	0.03	0.06	0.09	0.12	0.15	AVERAGE
1	AVG	SP D	.07482	.06259	.05927	.06482	.08037	.11005	.07542
	AVG	PESP	.05279	.07028	.09664	.08610	.07751	.06073	.07467
2	AVG	SP D	.07119	.06851	.08078	.09880	.12058	.14076	.09760
	AVG	PESP	.06199	.06532	.05860	.04485	.02537	.00079	.04282
3	AVG	SP D	.11597	.13075	.04736	.13401	.11882	.12208	.17372
	AVG	PESP	.03780	.02204	.00398	.00328	.01372	.00070	.091409
4	AVG	SP D	1.48787	.03280	.27756	.27492	.32545	.38008	.57986
	AVG	PESP	.74015	.78032	.02823	.04107	.02636	.00079	.00592
5	AVG	SP D	.39750	.01415	.43292	.45433	.47904	.50003	.44766
	AVG	PESP	.70147	.69363	.68549	.07713	.00065	.00001	.60110
6	AVG	SP D	.03134	.13841	.23169	.40571	.77928	3.70192	.80139
	AVG	PESP	.99996	.09919	.09025	.08732	.05739	.75241	.94876
ALL	AVG	SP D	.42078	.19170	.24093	.23876	.31720	.02002	.37594
	AVG	PESP	.88470	.09063	.09320	.09006	.07050	.03007	.87789
ORTHOGONAL COMBINATION DESIGN: 5 RADIAL AND 10 INNER VERTICES R2/R1= .340									
SURFACE	ERR	ST DEV	0.00	0.03	0.06	0.09	0.12	0.15	AVERAGE
1	AVG	SP D	.07680	.06480	.06238	.06983	.09034	.11002	.08046
	AVG	PESP	.05631	.07032	.09449	.08355	.07439	.05000	.07194
2	AVG	SP D	.07355	.06922	.07859	.09690	.12085	.14903	.09811
	AVG	PESP	.05950	.06480	.05877	.04455	.02332	.00009	.04109
3	AVG	SP D	.11610	.11001	.12695	.17850	.18918	.24008	.10273
	AVG	PESP	.03549	.03008	.01984	.07332	.08050	.07008	.00265
4	AVG	SP D	1.95748	.02093	.30031	.39835	.77338	2.54098	1.03089
	AVG	PESP	.70157	.77782	.82215	.77950	.76040	.74416	.77093
5	AVG	SP D	.40253	.01059	.43665	.45717	.48077	.50002	.45067
	AVG	PESP	.60002	.69203	.68481	.07740	.00090	.00003	.00093
6	AVG	SP D	.03137	.13072	.22695	.40503	.84100	5.15007	1.13238
	AVG	PESP	.99090	.09921	.09026	.09007	.04001	.45719	.89787
ALL	AVG	SP D	.44283	.19070	.20680	.20774	.41592	1.45005	.49587
	AVG	PESP	.83108	.09004	.09030	.07415	.00943	.76002	.80090
ORTHOGONAL COMBINATION DESIGN: 6 RADIAL AND 7 INNER VERTICES R2/R1= .189									
SURFACE	ERR	ST DEV	0.00	0.03	0.06	0.09	0.12	0.15	AVERAGE
1	AVG	SP D	.03310	.04161	.06563	.09004	.13341	.17920	.09153
	AVG	PESP	.03709	.08946	.08268	.09004	.09074	.09000	.09072
2	AVG	SP D	.04043	.05016	.06690	.08400	.10107	.11005	.07091
	AVG	PESP	.02495	.08103	.07216	.05935	.04261	.02107	.09030
3	AVG	SP D	.17009	.12581	.10072	.08731	.08178	.08217	.10944
	AVG	PESP	.06813	.07206	.07586	.07406	.07080	.06006	.09007
4	AVG	SP D	.11007	.10012	.09000	.08000	.08081	1.10000	.44927
	AVG	PESP	.02770	.01040	.03536	.05004	.02961	.04018	.05000
5	AVG	SP D	11.37097	1.06939	.09160	1.75308	1.21260	1.00004	2.90976
	AVG	PESP	.04045	.04001	.04738	.04780	.04835	.04005	.04763
6	AVG	SP D	.02067	.3.36308	.06662	.49103	.40109	.30044	.93559
	AVG	PESP	.09090	.07160	.05003	.09007	.08014	.09009	.04025
ALL	AVG	SP D	1.90130	.00166	.43259	.47032	.40180	.40003	.70008
	AVG	PESP	.01000	.00030	.07075	.09724	.08005	.08005	.08013

ORTHOGONAL COMBINATION DESIGN: 6 RADIAL AND 9 INNER VERTICES R2/R1 = .250								
SURFACE/ERR ST DEV		0.01	0.03	0.06	0.09	0.12	0.15	AVERAGE
1	AVG SP D	.03437	.04452	.06849	.10031	.13967	.18959	.09649
	AVG RESP	.94444	.98721	.98604	.96193	.92934	.87715	
2	AVG SP D	.04275	.05063	.06650	.08308	.10120	.11930	.07734
	AVG RESP	.98275	.97067	.96950	.95509	.93312	.91506	
3	AVG SP D	.17320	.12550	.10086	.08605	.06052	.08070	.10792
	AVG RESP	.96555	.97097	.98244	.98106	.97065	.96908	
4	AVG SP D	.11548	.15016	.53878	.40195	.50377	1.30001	.50279
	AVG RESP	.92235	.91020	.81402	.83056	.80400	.71597	
5	AVG SP D	2.02644	.98757	.96154	1.55075	1.31552	.99508	1.30565
	AVG RESP	.05583	.65086	.65093	.05904	.05920	.05907	
6	AVG SP D	.02257	3.04394	.99062	.51957	.42524	.37956	.97789
	AVG RESP	.99690	.85184	.96195	.97555	.98702	.98973	
ALL	AVG SP D	.40760	.81006	.45280	.45035	.42082	.51007	.51135
	AVG RESP	.91850	.89429	.87781	.89536	.88239	.85425	
ORTHOGONAL COMBINATION DESIGN: 6 RADIAL AND 9 INNER VERTICES R2/R1 = .287								
SURFACE/ERR ST DEV		0.01	0.03	0.06	0.09	0.12	0.15	AVERAGE
1	AVG SP D	.03960	.04536	.06713	.09734	.13539	.18575	.09477
	AVG RESP	.98594	.98035	.98030	.96509	.95380	.88503	
2	AVG SP D	.04443	.05064	.06572	.08245	.09953	.11709	.07664
	AVG RESP	.98509	.97763	.96935	.95000	.94013	.91521	
3	AVG SP D	.17154	.12452	.09165	.08300	.07759	.07701	.10563
	AVG RESP	.96724	.98056	.98450	.98304	.97020	.97002	
4	AVG SP D	.11623	.14507	.54322	.53390	.71965	1.18202	.54835
	AVG RESP	.91741	.91744	.80292	.78222	.76360	.72494	
5	AVG SP D	1.33979	.92329	.96794	2.56000	1.10702	.96000	1.31111
	AVG RESP	.06043	.66909	.66885	.06071	.06066	.06009	
6	AVG SP D	.02852	1.35516	17.02096	.76549	.54897	.47005	3.30596
	AVG RESP	.99098	.82425	.46171	.92043	.97192	.98009	
ALL	AVG SP D	.29003	.84034	3.12861	.69501	.44002	.49708	.91708
	AVG RESP	.91895	.89255	.81129	.88052	.87007	.85505	
ORTHOGONAL COMBINATION DESIGN: 6 RADIAL AND 10 INNER VERTICES R2/R1 = .314								
SURFACE/ERR ST DEV		0.01	0.03	0.06	0.09	0.12	0.15	AVERAGE
1	AVG SP D	.04280	.04092	.06770	.09584	.14005	.19442	.09847
	AVG RESP	.98104	.98031	.97955	.96199	.92030	.87211	
2	AVG SP D	.04609	.05161	.06722	.08511	.10349	.12201	.07937
	AVG RESP	.97877	.97092	.96744	.95411	.93013	.91502	
3	AVG SP D	.17300	.12338	.09611	.08057	.07390	.07578	.10347
	AVG RESP	.95097	.97964	.98456	.98345	.97067	.97591	
4	AVG SP D	.11711	.14767	1.16322	.50081	.60924	.67926	.54723
	AVG RESP	.91275	.91154	.79075	.77998	.75035	.71516	
5	AVG SP D	1.10704	.88117	1.00788	5.37971	.97532	.97914	1.72171
	AVG RESP	.07892	.67035	.67793	.07705	.07751	.07748	
6	AVG SP D	.02843	1.12042	10.83646	.74009	.51490	.43002	2.28083
	AVG RESP	.99090	.94033	.53002	.92003	.97130	.98009	
ALL	AVG SP D	.25244	.89020	2.20645	1.14000	.41282	.41430	.80518
	AVG RESP	.91857	.91196	.82454	.87967	.87507	.85499	
ORTHOGONAL COMBINATION DESIGN: 7 RADIAL AND 8 INNER VERTICES R2/R1 = .170								
SURFACE/ERR ST DEV		0.01	0.03	0.06	0.09	0.12	0.15	AVERAGE
1	AVG SP D	.03440	.05166	.08315	.12724	.18993	.28547	.12916
	AVG RESP	.98633	.97083	.95731	.91328	.83055	.72545	
2	AVG SP D	.03610	.05154	.07169	.09492	.12240	.15001	.08081
	AVG RESP	.98630	.97375	.95245	.92018	.87498	.81703	
3	AVG SP D	.14880	.23071	.12631	.09030	.09095	.09203	.13087
	AVG RESP	.97752	.96072	.94296	.95505	.93743	.91028	
4	AVG SP D	.11004	.72016	.40020	.45208	.03355	2.81300	.85870
	AVG RESP	.92699	.77474	.79706	.78040	.00010	.49944	
5	AVG SP D	1.00285	1.04141	1.15375	1.46275	3.35972	2.49003	1.75235
	AVG RESP	.04595	.64274	.63942	.05399	.03245	.02001	
6	AVG SP D	.02817	.23029	.21031	.20090	.20534	.20578	.10230
	AVG RESP	.99098	.99072	.99675	.99359	.99844	.99502	
ALL	AVG SP D	.20850	.78096	.34222	.40004	.70098	1.00002	.52370
	AVG RESP	.91319	.87258	.80132	.86732	.82467	.76572	
ORTHOGONAL COMBINATION DESIGN: 7 RADIAL AND 9 INNER VERTICES R2/R1 = .230								
SURFACE/ERR ST DEV		0.01	0.03	0.06	0.09	0.12	0.15	AVERAGE
1	AVG SP D	.03017	.05323	.08466	.12945	.19401	.29473	.13238
	AVG RESP	.98444	.97704	.95533	.91004	.83242	.72200	
2	AVG SP D	.03085	.05270	.07283	.09012	.12351	.15703	.09001
	AVG RESP	.98490	.97144	.94076	.91732	.87235	.81510	
3	AVG SP D	.15170	10.41102	.11051	.09011	.09339	.09411	2.83312
	AVG RESP	.98300	.97503	.95032	.95424	.93084	.91000	
4	AVG SP D	.11412	1.77017	.44484	.53310	.09053	3.87909	1.24120
	AVG RESP	.92278	.75713	.78974	.77350	.05410	.47907	
5	AVG SP D	.99709	1.08059	1.35174	3.01738	2.28994	1.30095	1.67178
	AVG RESP	.05701	.65064	.65016	.04057	.04080	.04023	
6	AVG SP D	.03010	.24776	.21447	.20000	.20730	.20742	.10002
	AVG RESP	.99090	.99054	.99061	.99044	.99020	.99014	
ALL	AVG SP D	.23104	3.27039	.38135	.08129	.59479	.98944	1.02508
	AVG RESP	.98045	.87220	.88232	.86702	.82281	.76077	

ORTHOGONAL COMBINATION DESIGN: 7 RADIAL AND 10 INNER VERTICES R2/R1= .275									
SURFACE/ERR ST DEV	0.00	0.03	0.06	0.09	0.12	0.15	AVERAGE		
1	AVG SP D	.03504	.05030	.07106	.14524	.23257	.40000	.16092	
	AVG RESP	.03760	.07366	.09767	.09054	.78355	.02001	.00773	
2	AVG SP D	.03505	.05031	.07107	.14525	.23257	.40000	.16092	
	AVG RESP	.03761	.07367	.09768	.09055	.78356	.02002	.00774	
3	AVG SP D	.22453	1.01471	1.13669	.11041	.10348	.10290	.41629	
	AVG RESP	.83214	.06519	.02954	.04298	.92850	.90346	.90863	
4	AVG SP D	.11790	.52760	.44546	.07250	.09411	21.33115	3.99813	
	AVG RESP	.91573	.76281	.77306	.72318	.00320	.43425	.70360	
5	AVG SP D	.99173	1.10519	1.30499	2.05240	3.43490	1.49508	1.84411	
	AVG RESP	.06670	.06050	.06014	.05070	.05310	.04903	.05830	
6	AVG SP D	.02505	.21290	.10901	.18579	.18000	.16705	.16480	
	AVG RESP	.99990	.09093	.99999	.99971	.99853	.99908	.99890	
ALL	AVG SP D	.24110	.62058	.38719	.04470	.03082	3.94074	1.11552	
	AVG RESP	.90549	.07195	.07527	.05302	.00257	.73100	.84012	
ORTHOGONAL COMBINATION DESIGN: 8 RADIAL AND 9 INNER VERTICES R2/R1= .167									
SURFACE/ERR ST DEV	0.00	0.03	0.06	0.09	0.12	0.15	AVERAGE		
1	AVG SP D	.03691	.05290	.07994	.11304	.17602	.26794	.12220	
	AVG RESP	.03669	.07210	.09082	.08909	.00450	.08928	.08088	
2	AVG SP D	.03693	.05292	.07771	.11067	.15710	.22009	.11028	
	AVG RESP	.03667	.07114	.09107	.09204	.01783	.72003	.08848	
3	AVG SP D	.03691	.05290	.06623	.08499	.11371	.15071	.06872	
	AVG RESP	.03694	.07066	.07331	.09570	.02640	.06221	.04554	
4	AVG SP D	.11327	1.30931	.58839	.49001	1.33200	2.53778	1.09656	
	AVG RESP	.93640	.76080	.77634	.74021	.07853	.54704	.74119	
5	AVG SP D	1.00638	1.03724	1.15392	1.01330	6.45870	1.05091	2.17139	
	AVG RESP	.04455	.63939	.63421	.02913	.02380	.01008	.03162	
6	AVG SP D	.02420	.17127	.15827	.15407	.15393	.15002	.13074	
	AVG RESP	.99990	.09947	.99955	.99956	.99950	.99955	.99961	
ALL	AVG SP D	.21223	.44048	.35406	.41321	1.43192	.86797	.62098	
	AVG RESP	.92055	.08779	.07803	.05204	.00840	.73906	.84789	
ORTHOGONAL COMBINATION DESIGN: 8 RADIAL AND 10 INNER VERTICES R2/R1= .225									
SURFACE/ERR ST DEV	0.00	0.03	0.06	0.09	0.12	0.15	AVERAGE		
1	AVG SP D	.03742	.05381	.07964	.11017	.16913	.25242	.11810	
	AVG RESP	.03694	.07079	.09295	.09373	.01521	.70009	.08605	
2	AVG SP D	.03752	.05413	.07711	.10735	.14791	.20006	.10493	
	AVG RESP	.03692	.06955	.09104	.09008	.02980	.74091	.08930	
3	AVG SP D	.03693	.05021	.06010	.07522	.09018	.12496	.07583	
	AVG RESP	.03374	.08438	.09025	.06708	.04210	.09701	.05932	
4	AVG SP D	.11276	2.41228	.92004	5.52147	.07917	3.90047	2.20883	
	AVG RESP	.92669	.79209	.77265	.74308	.06133	.55008	.74240	
5	AVG SP D	.03688	.08205	1.01569	1.10179	1.29710	1.91004	1.21707	
	AVG RESP	.05439	.64917	.64434	.03929	.03424	.02913	.04180	
6	AVG SP D	.02413	.17000	.16153	.15491	.15878	.15009	.14032	
	AVG RESP	.99998	.09945	.99952	.99951	.99947	.99943	.99956	
ALL	AVG SP D	.21084	.62125	.35582	1.18015	.42472	1.10431	.65454	
	AVG RESP	.92248	.09427	.08026	.05009	.01371	.75003	.85391	
ORTHOGONAL COMBINATION DESIGN: 9 RADIAL AND 10 INNER VERTICES R2/R1= .150									
SURFACE/ERR ST DEV	0.00	0.03	0.06	0.09	0.12	0.15	AVERAGE		
1	AVG SP D	.03593	.04694	.06094	.07815	.09925	.12003	.07448	
	AVG RESP	.03685	.07091	.06090	.03705	.00353	.05920	.03741	
2	AVG SP D	.03592	.04906	.07470	.11208	.16087	.25908	.11080	
	AVG RESP	.03685	.08004	.06451	.03355	.07054	.78073	.02087	
3	AVG SP D	.03567	.07116	.30750	.38034	.55337	.39403	.29936	
	AVG RESP	.07748	.06086	.07211	.79283	.77357	.78007	.00115	
4	AVG SP D	.11123	.12084	.14514	.17093	.20575	.25701	.10937	
	AVG RESP	.03210	.00080	.07360	.03170	.77930	.71912	.84048	
5	AVG SP D	1.00063	1.00907	1.70573	2.10203	1.24841	1.32405	1.41501	
	AVG RESP	.03339	.63012	.03279	.02730	.02191	.01007	.02999	
6	AVG SP D	.02821	.09004	.23725	.22300	.21967	.21998	.20316	
	AVG RESP	.00090	.00047	.00073	.00057	.00027	.00009	.00005	
ALL	AVG SP D	.21076	.28469	.42191	.51274	.41589	.43018	.37970	
	AVG RESP	.92112	.01103	.08370	.05302	.02553	.79006	.00476	
ORTHOGONAL COMBINATION DESIGN: 10 RADIAL AND 5 INNER VERTICES R2/R1= .081									
SURFACE/ERR ST DEV	0.00	0.03	0.06	0.09	0.12	0.15	AVERAGE		
1	AVG SP D	.03505	.04632	.05907	.07433	.09431	.05030	.06150	
	AVG RESP	.03507	.07701	.07799	.07052	.00088	.03022	.07075	
2	AVG SP D	.03504	.04633	.05903	.07430	.09419	.09137	.07719	
	AVG RESP	.03507	.07699	.07700	.07052	.00078	.03013	.06999	
3	AVG SP D	.12008	.30230	.07705	.13520	.12151	.12722	.25447	
	AVG RESP	.03505	.00000	.00001	.00037	.00063	.00005	.00000	
4	AVG SP D	.03505	.04632	.05903	.07433	.09419	.09137	.07719	
	AVG RESP	.03507	.07699	.07700	.07052	.00078	.03013	.06999	
5	AVG SP D	.03505	.04632	.05903	.07433	.09419	.09137	.07719	
	AVG RESP	.03507	.07699	.07700	.07052	.00078	.03013	.06999	
6	AVG SP D	.03505	.04632	.05903	.07433	.09419	.09137	.07719	
	AVG RESP	.03507	.07699	.07700	.07052	.00078	.03013	.06999	
ALL	AVG SP D	.22030	.20023	.00100	.23074	.31482	.40000	.07750	
	AVG RESP	.00001	.07374	.00000	.00000	.00000	.00000	.00000	

MINIMUM STRESS COEFFICIENT DESIGN : 5 RADIAL AND 9 INNER VERTICES R2/R1= .697									
SURFACEENERGY STRESS		0.00	0.03	0.06	0.09	0.12	0.15	AVERAGE	
1	AVG SP J	.97743	.97354	.96791	.96697	.97783	.99184	.97434	
	AVG RESP	.95064	.97389	.98295	.98435	.97637	.95615	.97199	
2	AVG SP J	.95058	.97455	.96151	.96532	.97171	.92531	.95560	
	AVG RESP	.95353	.95127	.95725	.95498	.93615	.92243	.94647	
3	AVG SP J	.95019	.95065	.91314	.94435	.95099	.92589	.92156	
	AVG RESP	.95153	.95449	.96665	.95459	.94879	.95367	.95464	
4	AVG SP J	.95024	.95075	.99352	.94453	.95608	.92607	.95573	
	AVG RESP	.95055	.94075	.92749	.91439	.93861	.94452	.94512	
5	AVG SP J	.95053	.97154	.94355	.94767	.95592	.92191	2.15506	
	AVG RESP	.95019	.94235	.95497	.95735	.95183	.95071	.95739	
6	AVG SP J	.95055	.93401	.95734	.97215	.95291	.99135	.99314	
	AVG RESP	.95498	.99453	.95925	.97935	.99605	.99760	.99682	
ALL	AVG SP J	.95034	.93121	.94273	1.14451	.95055	.94090	.93324	
	AVG RESP	.95064	.98040	.9710	.97597	.96864	.95418	.97374	
MINIMUM STRESS COEFFICIENT DESIGN : 5 RADIAL AND 7 INNER VERTICES R2/R1= .607									
SURFACEENERGY STRESS		0.00	0.03	0.06	0.09	0.12	0.15	AVERAGE	
1	AVG SP J	.95005	.97357	.97543	.97753	.98643	.95344	.98543	
	AVG RESP	.95000	.95755	.97514	.97551	.97994	.95741	.96608	
2	AVG SP J	.95035	.95453	.94331	.94435	.93448	.91168	.93010	
	AVG RESP	.95067	.95534	.95013	.95045	.92480	.90716	.93010	
3	AVG SP J	.95044	.95153	.95242	.95025	.92098	.91450	.92212	
	AVG RESP	.95122	.95454	.95563	.95000	.95025	.92007	.95362	
4	AVG SP J	.95075	5.19528	.95457	.95451	.95227	.95326	1.81416	
	AVG RESP	.95059	.97104	.94974	.97651	.98372	.95410	.95547	
5	AVG SP J	.97142	.95298	1.19783	4.47239	2.33101	3.01084	2.93191	
	AVG RESP	.95432	.94298	.93954	.93848	.93885	.94749	.95951	
6	AVG SP J	.95043	.93438	.97139	.92825	.92472	.94849	.92328	
	AVG RESP	.99999	.99950	.99852	.99522	.99116	.97954	.99415	
ALL	AVG SP J	.95038	1.57576	.98433	1.74427	.95598	.91587	.88276	
	AVG RESP	.95095	.97349	.97996	.97321	.96314	.95520	.95782	
MINIMUM STRESS COEFFICIENT DESIGN : 5 RADIAL AND 8 INNER VERTICES R2/R1= .613									
SURFACEENERGY STRESS		0.00	0.03	0.06	0.09	0.12	0.15	AVERAGE	
1	AVG SP J	.95131	.95567	.95691	.96236	.97897	.99787	.97578	
	AVG RESP	.94749	.97505	.98727	.98836	.98371	.97588	.97646	
2	AVG SP J	.95097	.97182	.96325	.97567	.97046	.91388	.95468	
	AVG RESP	.95021	.95229	.96371	.96067	.95556	.94941	.95697	
3	AVG SP J	.95074	.95730	.95609	.93951	.95832	.95438	.94991	
	AVG RESP	.95023	.95699	.96184	.93735	.93020	.92723	.94131	
4	AVG SP J	1.27073	.95281	2.58759	2.09471	.93702	.93429	1.12419	
	AVG RESP	.95040	.97515	.96581	.93916	.95954	.97833	.97162	
5	AVG SP J	.94329	4.23092	2.59475	2.65251	2.34597	1.68577	2.39470	
	AVG RESP	.94994	.93145	.95277	.94369	.93046	.91993	.97962	
6	AVG SP J	.95032	.93494	.97522	.97829	.923157	.92100	.91789	
	AVG RESP	.99998	.99913	.99792	.99674	.99570	.99480	.99738	
ALL	AVG SP J	.95199	.95541	.93647	.93519	.95959	.94120	.93503	
	AVG RESP	.95474	.95775	.95822	.95948	.95919	.95944	.95981	
MINIMUM STRESS COEFFICIENT DESIGN : 5 RADIAL AND 9 INNER VERTICES R2/R1= .614									
SURFACEENERGY STRESS		0.00	0.03	0.06	0.09	0.12	0.15	AVERAGE	
1	AVG SP J	.95053	.95826	.95720	.95643	.96236	.97293	.96448	
	AVG RESP	.94426	.97141	.97414	.97413	.98926	.98633	.97775	
2	AVG SP J	.95064	.97415	.97405	.98432	.98483	.95055	.98758	
	AVG RESP	.94490	.95445	.97223	.97415	.97276	.96930	.96705	
3	AVG SP J	.95081	.98475	.97415	.97201	.97081	.92625	1.9996	
	AVG RESP	.95126	.93594	.95133	.96646	.96218	.95242	.94820	
4	AVG SP J	3.00070	.93347	.91051	.91534	.91838	.97472	1.30360	
	AVG RESP	.95067	.95375	.95219	.95454	.95091	.92444	.97896	
5	AVG SP J	2.39286	3.72431	24.26737	2.69508	2.24639	1.99504	7.87033	
	AVG RESP	.95009	.95259	.95688	.95659	.95433	.95163	.95606	
6	AVG SP J	.95022	.92507	.95410	.94425	.95688	.92986	.95023	
	AVG RESP	.95043	.99930	.99940	.99756	.99684	.99522	.99905	
ALL	AVG SP J	1.55833	.95067	5.00223	.95067	.98328	.94565	1.61186	
	AVG RESP	.95055	.95064	.97414	.97175	.97771	.97723	.97751	
MINIMUM STRESS COEFFICIENT DESIGN : 5 RADIAL AND 10 INNER VERTICES R2/R1= .620									
SURFACEENERGY STRESS		0.00	0.03	0.06	0.09	0.12	0.15	AVERAGE	
1	AVG SP J	.95031	.97215	.97065	.97117	.97931	.98799	.97478	
	AVG RESP	.94417	.97096	.97092	.97085	.98254	.97433	.97106	
2	AVG SP J	.95067	.97401	.97553	.98417	.98416	.95001	.99158	
	AVG RESP	.94473	.95095	.97331	.97391	.97018	.95356	.95578	
3	AVG SP J	.95053	.97447	.97011	.97474	.97092	.92406	.94247	
	AVG RESP	.95024	.95737	.95006	.95075	.92841	.95007	.94642	
4	AVG SP J	2.94415	1.32522	.95777	.92431	.92015	.92014	.95627	
	AVG RESP	.95024	.95915	.95064	.92539	.95889	.97654	.96665	
5	AVG SP J	2.35015	2.28092	2.00330	2.15342	1.88775	2.14895	2.23825	
	AVG RESP	.95099	.95962	.95004	.95098	.95739	.95669	.95429	
6	AVG SP J	.95013	.92503	.95317	.97098	.93021	.92572	.91742	
	AVG RESP	.95004	.99940	.99937	.99705	.99557	.99430	.99740	
ALL	AVG SP J	.95069	.95072	.95021	.95035	.94725	.94945	.95086	
	AVG RESP	.94439	.95092	.97215	.98419	.98543	.98071	.97777	

[illegible]

MINIMUM CLASS CO-BI RATIO DESIGN: 6 RADIAL AND 10 TAPER VERTICES R2/R1= .615									
SURFACEGENERATOR		0.10	0.03	0.01	0.00	0.12	0.15	AVERAGE	
1	AVG SP	.07354	.07345	.06505	.11521	.14159	.18925	.11397	
	AVG RECP	.06000	.07388	.07149	.06000	.03267	.09351	.09010	
2	AVG SP	.07042	.05935	.06422	.07946	.09722	.11588	.08109	
	AVG RECP	.06037	.07146	.06923	.06111	.04839	.03145	.05775	
3	AVG SP	.06051	2.27115	.03319	.29959	.34670	.20936	.62007	
	AVG RECP	.01071	.04100	.00092	.04059	.07928	.09703	.06326	
4	AVG SP	.01060	.05313	.01051	.34005	.50609	.77783	.35681	
	AVG RECP	.01071	.04073	.00204	.74001	.64752	.59080	.77062	
5	AVG SP	.06735	.07011	1.00037	1.17248	.79193	.05329	.95345	
	AVG RECP	.02131	.02010	.02311	.02435	.02543	.02750	.02003	
6	AVG SP	.02012	.07059	.04091	.05122	1.64516	5.63435	1.41373	
	AVG RECP	.00099	.09084	.00411	.07725	.04479	.36733	.07205	
ALL	AVG SP	.06013	.06931	.07594	.14176	.56812	1.29733	.50777	
	AVG RECP	.01275	.00132	.00132	.06007	.03004	.07347	.06614	
MINIMUM CLASS CO-BI RATIO DESIGN: 7 RADIAL AND 5 TAPER VERTICES R2/R1= .637									
SURFACEGENERATOR		0.10	0.03	0.01	0.00	0.12	0.15	AVERAGE	
1	AVG SP	.06173	.06125	.06157	.02033	.07563	4.62010	1.01062	
	AVG RECP	.07771	.05324	.07003	.03342	.03035	.01506	.07149	
2	AVG SP	.06135	.07481	.01707	.05309	.01947	.02069	.05591	
	AVG RECP	.07735	.05090	.00000	.02551	.071751	.05931	.06440	
3	AVG SP	.06701	.02044	.04177	1.02275	.54201	.36710	.47735	
	AVG RECP	.04040	.07005	.03010	.06039	.02540	.07351	.04349	
4	AVG SP	.01074	1.73092	1.00007	0.05533	.67933	.41095	1.73385	
	AVG RECP	.00045	.04034	.01052	.04045	.07895	.07370	.06590	
5	AVG SP	1.20075	1.65225	3.00021	4.00034	1.71068	1.27015	2.25666	
	AVG RECP	.07333	.07247	.07163	.07030	.06999	.06920	.07124	
6	AVG SP	.02775	.07094	.03064	.02339	.02605	.03048	.02036	
	AVG RECP	.09099	.09035	.09091	.09094	.09089	.09087	.090750	
ALL	AVG SP	.06097	.06026	.07025	1.05062	.71570	1.22093	.97396	
	AVG RECP	.09719	.04050	.02053	.07556	.07002	.06006	.070150	
MINIMUM CLASS CO-BI RATIO DESIGN: 7 RADIAL AND 6 TAPER VERTICES R2/R1= .663									
SURFACEGENERATOR		0.10	0.03	0.01	0.00	0.12	0.15	AVERAGE	
1	AVG SP	.06049	.07094	.02095	.02001	.03104	.03358	.02017	
	AVG RECP	.07435	.06105	.01377	.03348	.07020	.03004	.03004	
2	AVG SP	.06055	.06296	.03022	.01000	.04795	.03046	.03065	
	AVG RECP	.07490	.05751	.02723	.04039	.03052	.03052	.03052	
3	AVG SP	.06055	.02429	.01024	.04052	.03127	.03002	.03002	
	AVG RECP	.01005	.02935	.01000	.04052	.03000	.03000	.03000	
4	AVG SP	.01031	.02072	.01000	0.16106	0.47005	1.51020	1.50628	
	AVG RECP	.01058	.07010	.01000	.04055	.03151	.03107	.03000	
5	AVG SP	.06049	.03095	1.01002	1.40041	0.67045	1.40197	12.07012	
	AVG RECP	.06055	.06055	.06050	.06186	.07039	.06075	.06097	
6	AVG SP	.02775	.07031	.02040	.02051	.02724	.02710	.02014	
	AVG RECP	.09099	.09099	.09099	.09099	.09099	.09099	.09099	
ALL	AVG SP	.06051	.03094	.02031	.03001	1.06050	.06049	2.56076	
	AVG RECP	.09099	.07135	.02031	.07039	.07039	.07039	.07039	
MINIMUM CLASS CO-BI RATIO DESIGN: 7 RADIAL AND 7 TAPER VERTICES R2/R1= .600									
SURFACEGENERATOR		0.10	0.03	0.01	0.00	0.12	0.15	AVERAGE	
1	AVG SP	.06039	.06065	.06016	.06042	.02060	.03047	.03047	
	AVG RECP	.07001	.06002	.06011	.07097	.07062	.06075	.07035	
2	AVG SP	.06013	.06027	.07075	.07044	.02012	.01537	.01067	
	AVG RECP	.07001	.06002	.06007	.09007	.06000	.06000	.06000	
3	AVG SP	0.01045	.06004	0.01039	.06003	.01000	.01037	1.08035	
	AVG RECP	.07001	.06002	.06002	.09007	.06000	.06000	.06000	
4	AVG SP	.01049	.02030	.07041	.03031	1.70019	1.01078	2.70092	
	AVG RECP	.06001	.03000	.06001	.06000	.06000	.06000	.06000	
5	AVG SP	.06020	.01069	.06000	.06000	.06000	.06000	.06000	
	AVG RECP	.06001	.06000	.06000	.06000	.06000	.06000	.06000	
6	AVG SP	.02033	.02033	.02033	.02033	.02033	.02033	.02033	
	AVG RECP	.06001	.06000	.06000	.06000	.06000	.06000	.06000	
ALL	AVG SP	1.01015	.06015	1.01004	.06007	.06000	.06000	1.02030	
	AVG RECP	.06001	.06000	.06000	.06000	.06000	.06000	.06000	
MINIMUM CLASS CO-BI RATIO DESIGN: 7 RADIAL AND 8 TAPER VERTICES R2/R1= .600									
SURFACEGENERATOR		0.10	0.03	0.01	0.00	0.12	0.15	AVERAGE	
1	AVG SP	.06039	.06015	.06015	.06004	.02047	.03002	.03002	
	AVG RECP	.07001	.06017	.06000	.06000	.07000	.07000	.07000	
2	AVG SP	.06033	.06000	.07000	.06000	.01000	.01000	.01000	
	AVG RECP	.07001	.06000	.06000	.06000	.06000	.06000	.06000	
3	AVG SP	.01044	.02015	2.01043	.03017	.03000	.03000	.03000	
	AVG RECP	.06017	.02043	.06015	.06000	.06000	.06000	.06000	
4	AVG SP	.02028	.02012	2.01000	1.01000	1.01000	1.01000	1.01000	
	AVG RECP	.06013	.01078	.06000	.06000	.06000	.06000	.06000	
5	AVG SP	.06013	.06000	1.01000	1.01000	1.01000	1.01000	1.01000	
	AVG RECP	.06001	.06000	.06000	.06000	.06000	.06000	.06000	
6	AVG SP	.02033	.02033	.02033	.02033	.02033	.02033	.02033	
	AVG RECP	.06001	.06000	.06000	.06000	.06000	.06000	.06000	
ALL	AVG SP	1.01004	.06017	3.01000	.06000	.06000	1.01000	1.01030	
	AVG RECP	.06001	.07000	.06000	.06000	.06000	.06000	.06000	

MINIMUM STIAS CO-BINATION DESIGN: 10 RADIAL AND 9 INNER VERTICES R2/R1= 1.005									
SURFACEENERGY ST DEV									
		0.00	0.03	0.06	0.09	0.12	0.15	AVERAGE	
1	AVG SP D	.98696	.98764	.98917	.11915	.16086	.21674	.11407	
	AVG RESP	.97463	.97192	.96331	.90777	.87038	.80036	.91607	
2	AVG SP D	.99581	.99604	.99642	.15775	.27095	.52458	.19359	
	AVG RESP	.97963	.97365	.96562	.90971	.81663	.60914	.87391	
3	AVG SP D	.91284	.97636	.96255	.817091	.96778	.31657	1.58585	
	AVG RESP	.99555	.98599	.97263	.92762	.78285	.87759	.78524	
4	AVG SP D	.10026	.09900	.11859	.14799	.18503	.23766	.14692	
	AVG RESP	.92301	.90513	.87362	.83454	.79142	.74730	.84584	
5	AVG SP D	1.00453	2.79579	1.44416	1.01135	.98942	1.09270	1.39550	
	AVG RESP	.97387	.96741	.96387	.96024	.95651	.95267	.96193	
6	AVG SP D	.92785	.79731	.81601	.31498	.27545	.25656	.36553	
	AVG RESP	.92799	.93003	.93169	.92980	.92409	.92493	.92213	
ALL	AVG SP D	.21087	.64353	.49182	1.65271	.38025	.44030	.63358	
	AVG RESP	.92473	.90569	.87179	.76736	.81865	.79133	.84502	
MINIMUM STIAS CO-BINATION DESIGN: 10 RADIAL AND 7 INNER VERTICES R2/R1= 1.534									
SURFACEENERGY ST DEV									
		0.00	0.03	0.06	0.09	0.12	0.15	AVERAGE	
1	AVG SP D	.95791	.95701	.95925	.16171	.24922	.41090	.17517	
	AVG RESP	.97771	.95415	.93084	.86546	.75538	.60907	.85743	
2	AVG SP D	.95592	.97926	.95172	.24973	.31697	3.33516	.80228	
	AVG RESP	.97771	.97277	.95503	.87499	.37785	.21455	.71416	
3	AVG SP D	.95352	.92120	5.63998	.24295	.17583	.15275	1.12256	
	AVG RESP	.99095	.76791	.93467	.91344	.95036	.95724	.86344	
4	AVG SP D	.10221	.10669	.13584	.18233	.25834	.35171	.18785	
	AVG RESP	.91229	.89090	.86243	.79236	.71532	.64169	.79852	
5	AVG SP D	.91340	1.74767	1.24066	.93693	1.04323	11.07538	2.83903	
	AVG RESP	.67359	.67560	.67253	.66736	.66613	.66274	.67184	
6	AVG SP D	.92781	4.49725	.30943	.29125	.22517	.21801	.91732	
	AVG RESP	.99999	.61477	.92464	.90728	.99676	.99499	.93324	
ALL	AVG SP D	.20353	1.18650	1.23456	.34233	.47679	2.59240	1.00603	
	AVG RESP	.92494	.91420	.87387	.83940	.74364	.68039	.80510	
MINIMUM STIAS CO-BINATION DESIGN: 10 RADIAL AND 5 INNER VERTICES R2/R1= 1.554									
SURFACEENERGY ST DEV									
		0.00	0.03	0.06	0.09	0.12	0.15	AVERAGE	
1	AVG SP D	.95949	.95463	.95773	.13571	.19448	.28313	.13771	
	AVG RESP	.97597	.96587	.94172	.89700	.82318	.71720	.86682	
2	AVG SP D	.95949	.97554	.92172	.21235	.43405	1.99284	.49133	
	AVG RESP	.97597	.97390	.95243	.88051	.66729	.22385	.77897	
3	AVG SP D	.91473	.37312	.80549	.31747	.18969	.16040	.40948	
	AVG RESP	.80617	.93501	.91925	.87678	.94620	.95484	.87438	
4	AVG SP D	.10512	.10263	.12571	.16218	.21117	.27465	.16429	
	AVG RESP	.91590	.39342	.95543	.80720	.75298	.69487	.81798	
5	AVG SP D	.85332	1.05605	15.68283	1.11267	.92274	.98283	3.43408	
	AVG RESP	.68342	.68280	.68011	.67735	.67452	.67162	.67463	
6	AVG SP D	.92774	21.33121	.36884	.26475	.23720	.22519	3.82578	
	AVG RESP	.99999	.58183	.99074	.99093	.99595	.99639	.92564	
ALL	AVG SP D	.32332	3.91808	2.72684	.36637	.36499	.65217	1.40961	
	AVG RESP	.89324	.82546	.87328	.85564	.81035	.71047	.82924	
MINIMUM STIAS CO-BINATION DESIGN: 10 RADIAL AND 3 INNER VERTICES R2/R1= 1.569									
SURFACEENERGY ST DEV									
		0.00	0.03	0.06	0.09	0.12	0.15	AVERAGE	
1	AVG SP D	.95763	.95825	.97473	.09803	.12601	.15876	.09558	
	AVG RESP	.97434	.96940	.95685	.93620	.90647	.86596	.83489	
2	AVG SP D	.95763	.96764	.95913	.12629	.18621	.28636	.13564	
	AVG RESP	.97434	.97651	.96894	.94422	.89653	.79397	.92712	
3	AVG SP D	.39073	1.16789	.92932	11.33355	.89713	.24358	2.41506	
	AVG RESP	.87617	.45559	.97265	.72741	.73242	.95073	.82493	
4	AVG SP D	.10843	.09510	.16942	.13354	.16017	.18048	.13273	
	AVG RESP	.91270	.90092	.97725	.84027	.81615	.78744	.85529	
5	AVG SP D	.80191	.99394	1.36242	2.11811	1.21315	1.06360	1.30204	
	AVG RESP	.69159	.65080	.96067	.93324	.68036	.67743	.68459	
6	AVG SP D	.92774	.37715	1.33155	.34034	.30279	.27761	.44986	
	AVG RESP	.99999	.98383	.97735	.94776	.90229	.99384	.97351	
ALL	AVG SP D	.25004	.44212	.91534	2.35529	.60790	.37510	.75532	
	AVG RESP	.90457	.89585	.97323	.95003	.83743	.83189	.86689	
MINIMUM STIAS CO-BINATION DESIGN: 10 RADIAL AND 10 INNER VERTICES R2/R1= 1.481									
SURFACEENERGY ST DEV									
		0.00	0.03	0.06	0.09	0.12	0.15	AVERAGE	
1	AVG SP D	.95942	.95640	.97105	.09434	.11340	.13735	.08934	
	AVG RESP	.97292	.96434	.95843	.90239	.81099	.69127	.89246	
2	AVG SP D	.95942	.95941	.98134	.11139	.15843	.21786	.11531	
	AVG RESP	.97292	.97569	.97095	.95335	.91872	.85499	.94194	
3	AVG SP D	.27553	.27273	.36123	.90941	2.77228	.77364	.91147	
	AVG RESP	.96591	.97755	.96581	.91561	.68243	.72794	.80482	
4	AVG SP D	.11137	.09242	.10801	.13724	.15268	.17526	.12342	
	AVG RESP	.96593	.90115	.96959	.94611	.82972	.80743	.83328	
5	AVG SP D	.85714	.95120	.91733	1.13761	2.62083	1.81615	1.36704	
	AVG RESP	.69713	.69405	.69083	.69766	.68440	.68109	.68921	
6	AVG SP D	.92777	.34133	5.1777	.50001	.36036	.31223	.99436	
	AVG RESP	.99999	.99254	.97345	.94362	.90901	.94100	.94521	
ALL	AVG SP D	.25217	.27079	.36745	.90351	1.90510	.57001	.88516	
	AVG RESP	.96510	.90130	.95170	.92702	.83405	.92505	.86532	

		HATHOR PIAS CENTRAL COMPOSITE DESIGN						
SURFACE/ENERGY DEV		0.00	0.03	0.06	0.09	0.12	0.15	AVERAGE
1	AVG SP U	.03443	.03795	.03549	.02742	.00695	.03496	.06920
	AVG RESP	.98342	.98579	.98106	.97079	.97019	.96421	.97758
2	AVG SP U	.03447	.04105	.05645	.07090	.00367	.09529	.06366
	AVG RESP	.98340	.99124	.98857	.98370	.97765	.97065	.98137
3	AVG SP U	.06031	.06753	.07597	.08401	.09194	.09997	.07992
	AVG RESP	.95091	.95545	.95161	.94764	.94345	.93957	.94968
4	AVG SP U	.11340	.09335	.16296	.11277	.12025	.12641	.11119
	AVG RESP	.93420	.96091	.94310	.93797	.93201	.92521	.93457
5	AVG SP U	1.04370	1.04332	1.0205	1.04036	1.03978	1.04034	1.04267
	AVG RESP	.63348	.63031	.63015	.64051	.64285	.64520	.63933
6	AVG SP U	.02328	.23016	.17373	5.44338	3.23287	1.19165	1.85701
	AVG RESP	.99498	.99050	.98902	.97443	.97917	.98358	.97896
ALL	AVG SP U	.22006	.25433	.31944	1.21097	.77591	.44144	.53711
	AVG RESP	.91756	.91005	.91525	.89334	.79089	.88440	.87908
		HATHOR PIAS CENTRAL COMPOSITE DESIGN						
SURFACE/ENERGY DEV		0.00	0.03	0.06	0.09	0.12	0.15	AVERAGE
1	AVG SP U	.03443	.03791	.07042	.11213	.15033	.30795	.12495
	AVG RESP	.98342	.98346	.97999	.96994	.95983	.75696	.92477
2	AVG SP U	.03447	.04313	.05953	.11603	.20273	.40183	.14464
	AVG RESP	.98340	.98743	.97939	.95426	.88403	.69675	.91504
3	AVG SP U	.06031	.05520	.10330	.44951	.92095	1.74755	.50877
	AVG RESP	.95091	.96814	.94806	.79579	.76966	.77488	.86824
4	AVG SP U	.11340	.14305	.90980	.49346	.42058	.49502	.35622
	AVG RESP	.93420	.90274	.80591	.76050	.76025	.69510	.80995
5	AVG SP U	1.04370	1.12385	1.36031	2.54971	2.65521	1.41758	1.69223
	AVG RESP	.63348	.63063	.62767	.62461	.62144	.61316	.62600
6	AVG SP U	.02328	.18411	3.17649	.43012	.30883	.27100	.71680
	AVG RESP	.99498	.99445	.98354	.97880	.96903	.99187	.93962
ALL	AVG SP U	.22006	.25412	.57281	.74201	.76577	.77383	.60727
	AVG RESP	.91756	.91014	.83659	.84382	.81905	.75745	.84744
		HATHOR PIAS CENTRAL COMPOSITE DESIGN						
SURFACE/ENERGY DEV		0.00	0.03	0.06	0.09	0.12	0.15	AVERAGE
1	AVG SP U	.03443	.05073	.17996	3.09128	1.07653	.37592	.80364
	AVG RESP	.98342	.98356	.92013	.25972	.40555	.70396	.71056
2	AVG SP U	.03447	.06110	.16188	.65615	3.62677	.79314	.39893
	AVG RESP	.98340	.98017	.90996	.94328	.13015	.47425	.67104
3	AVG SP U	.06031	.07293	.17092	.41630	.41307	1.58562	.45814
	AVG RESP	.95090	.96159	.9524	.79956	.77157	.75186	.85514
4	AVG SP U	.11340	.33977	.33277	1.44402	.71739	.40207	.65040
	AVG RESP	.93420	.81437	.70558	.65761	.60681	.73003	.75677
5	AVG SP U	1.04369	1.06912	1.08873	1.11071	1.13432	1.15987	1.10174
	AVG RESP	.63350	.62956	.62559	.62157	.61753	.61345	.62353
6	AVG SP U	.02328	1.51592	.28013	.24264	.23035	.22479	.42135
	AVG RESP	.99493	.95459	.99212	.99549	.99631	.99667	.97319
ALL	AVG SP U	.22006	.51075	.37006	1.25218	1.19974	.75740	.71987
	AVG RESP	.91757	.97159	.85644	.64454	.58832	.71170	.76504

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